Can Risk Premium Explain the Uncovered Interest Parity Puzzle?
A Nonparametric Approach

Jing Li

Christopher J. Adams

Miami University
Uncovered Interest Parity Puzzle

1. Uncovered Interest Parity (UIP)—Condition of No Arbitrage

2. Comparable assets should yield same expected returns after adjustment of exchange rates

3. However, historical data usually support the success of the strategy of investing in the asset with rising interest rate, regardless of exchange rate

4. Why? Basically factors other than exchange rate can adjust to equalize expected returns
Omitted Variable

1. When regressing the change in exchange rate onto interest differential, the beta coefficient is typically less than 1, and sometimes even negative.

2. Currency with rising (falling) interest rate does not depreciate (appreciate) enough as indicated by UIP.

3. One way to explain the downward bias in beta coefficient is recognition of omitted variable—risk premium (Fama 1984).

4. But what is risk premium? Is it a “catch-all” variable?
Fama’s Explanation

1. Fama (1984) shows that (P1) a negative covariance between the risk premium and interest differential can lead to a downward bias; (P2) to explain a negative slope coefficient, that covariance has to be less than the negative variance of interest differential

2. The main challenge to empirically verify Fama’s theory is the lack of observable risk premium
Contribution of This Paper

1. We suggest two nonparametric estimates of risk premium
2. We use the estimated risk premium to check Fama’s explanation
Identification Strategy: Smoothness of Risk Premium

1. By nature, the risk premium is unobserved since it depends on personal perceived risk and attitude toward the risk.

2. Consider a “structural model” (first order condition for the generalized Lucas (1978) intertemporal asset pricing model) of the risk premium $\theta_t$:

$$E(\theta_t | \Omega_t) = \frac{\text{cov} \left( \frac{u'(c_{t+1})}{p_{t+1}} \frac{p_t}{u'(c_t)}, \frac{S_{t+1}-S_t}{S_t} | \Omega_t \right)}{-E \left( \frac{u'(c_{t+1})}{p_{t+1}} \frac{p_t}{u'(c_t)} | \Omega_t \right)}$$  (1)

where $u$ represents the utility function, $c$ is consumption, $p$ is price, $S_t$ is the exchange rate (domestic currency per one FX), and $\Omega$ is the information set.

3. There are two sources of smoothness for risk premium

   (a) For individual—consumption smoothing, sticky price and time-invariant utility function

   (b) For population—central limit theorem (applied to average)
Parametric Estimate of Risk Premium

Restrictive assumptions are needed to obtain a structural form estimate

1. Constant Relative Risk Aversion (CRRA) utility
2. Information set contains all relevant information
3. The economic agent is rational

For the sake of robustness and flexibility, this paper instead adopts a nonparametric or “reduced-form” methodology
Nonparametric Estimate of Risk Premium

1. We apply the Hodrick-Prescott (HP) filter (Hodrick and Prescott 1997) to the Fama regression forecast error with the UIP imposed, and extracting the slow-evolving trend component as the estimate for the risk premium utility

2. Fourier Flexible Form with a small number of low frequency components (Gallant 1981) is employed to obtain a similar estimate of the risk premium

3. Our nonparametric approach makes much weaker assumptions relative to the structural-form estimate
Fourier Flexible Form

1. In theory the Fourier series with trigonometric terms is able to approximate any absolutely integrable function.

2. In that regard, the Fourier method is even more flexible than the HP filter as smoothness is not needed.

3. Despite that difference, we show that the risk premium estimated by the FFF with a small number of frequencies is very similar to that generated by the HP filter.

4. This finding indicates that the HP trend is unlikely to be spurious, thus a major concern raised by Hamilton (2018) is resolved (i.e., we focus on smoothness of the trend, not a stationary business cycle component).
Fama Regression

Let $ID_t$ denote the home minus foreign nominal interest rate differential, and $\Delta s_{t+1} = \log S_{t+1} - \log S_t$ be the one-period change in log spot exchange rate. Fama regression is

$$\Delta s_{t+1} = \beta_0 + \beta_1 ID_t + u_t$$  \hspace{1cm} (2)

Testing UIP hypothesis is equivalent to testing

$$H_0 : \beta_1 = 1$$ \hspace{1cm} (3)

which states that, ceteris paribus, the interest differential is an unbiased predictor for the change in spot exchange rate
The UIP Forecasting Error Decomposition

We first treat the UIP as a forecasting error decomposition

\[ \Delta s_{t+1} = ID_t + u_t \]  \hspace{1cm} (4)

\[ u_t = \theta_t + e_t \]  \hspace{1cm} (5)

1. The interest differential is the part of exchange rate change explained by the UIP, whereas the error term is the unexplained part

2. (5) divides the composite forecast error \( u_t \) into the risk premium \( \theta_t \) (signal) and an idiosyncratic error term \( e_t \) (noise)
**HP Filter Estimate**

1. We apply the HP filter to the observable composite forecast error

\[ u_t \equiv \Delta s_{t+1} - ID_t \]  

and extract the slow-evolving smooth component, which serves as the estimate for unobservable risk premium

2. More explicitly, the HP filter aims to solve this optimization problem:

\[
\min_{\hat{\theta}_t^{HP}} \sum_{t=1}^{n} (u_t - \hat{\theta}_t^{HP})^2 + \lambda \sum_{t=2}^{n-1} [(\hat{\theta}_{t+1}^{HP} - \hat{\theta}_t^{HP}) - (\hat{\theta}_t^{HP} - \hat{\theta}_{t-1}^{HP})]^2
\]

where \( \hat{\theta}_t^{HP} \) is the HP estimate of the risk premium (trend component)
**FFF Estimate of Risk Premium**

1. Consider replacing the risk premium $\theta_t$ with its Fourier approximation $FFF_t$

   $$ u_t = FFF_t + \hat{e}_t $$  (8)

   $$ FFF_t = \mu + \sum_{k=1}^{p} \alpha_k \cos\left(\frac{2\pi kt}{n}\right) + \sum_{k=1}^{p} \beta_k \sin\left(\frac{2\pi kt}{n}\right) $$  (9)

   where $k$ is frequency index, $p$ is the number of frequencies, and $n$ is sample size.

2. When $p = 0$, the risk premium is assumed to be a constant. With rising $p$ we can allow for increasingly complex patterns in the risk premium. When $p = n/2$, the Fourier approximation is perfect for any absolutely integrable function.

3. FFF method is really a “data-driven” or “endogenous” way of estimating the unknown risk premium.
More about FFF

1. It is noteworthy that the FFF approximation provides adequate approximation for not just smooth functions, but also functions with jumps or kinks

2. See Panels A, B, C, D of Figure 1 in Enders and Li (2015) for an illustration

3. Basically, those authors show that the FFF works well even in the absence of smoothness in the target function. Consequently we have more faith in the trend of HP filter if it shows similarity to the Fourier approximation
More about FFF

Four Simulated Series and U.S. GDP

Panel A: Smooth Reinforcing Breaks
Panel B: Smooth Offsetting Breaks
Panel C: Sharp Reinforcing Breaks
Panel D: Sharp Offsetting Breaks
Panel E: Log Potential GDP
Panel F: Log U.S. GDP
## Summary Statistics

### Table 1: Summary Statistics (1971:1-2016:6)

<table>
<thead>
<tr>
<th>Series</th>
<th>$n$</th>
<th>Mean</th>
<th>SD</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i^{UK}$</td>
<td>546</td>
<td>6.904</td>
<td>4.229</td>
<td>0.224</td>
<td>16.181</td>
</tr>
<tr>
<td>$i^{CA}$</td>
<td>546</td>
<td>6.044</td>
<td>4.198</td>
<td>0.170</td>
<td>20.820</td>
</tr>
<tr>
<td>$i^{US}$</td>
<td>546</td>
<td>4.876</td>
<td>3.439</td>
<td>0.010</td>
<td>16.290</td>
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<tr>
<td>$S^{UK}$</td>
<td>546</td>
<td>1.757</td>
<td>0.308</td>
<td>1.093</td>
<td>2.618</td>
</tr>
<tr>
<td>$S^{CA}$</td>
<td>546</td>
<td>0.839</td>
<td>0.115</td>
<td>0.625</td>
<td>1.047</td>
</tr>
</tbody>
</table>

Note: $i$ is the treasury bill rate, $S$ is the spot exchange rate; UK, CA, US stand for United Kingdom, Canada, and United States, respectively.
Fama Regression

Figure 1: Time-Varying Beta1

Panel A: UK

Panel B: Canada
Figure 2: Composite Forecast Error and HP Risk Premium

Panel A: UK, u

Panel B: UK, thetha

Panel C: Canada, u

Panel D: Canada, thetha
Comparing HP and FFF Risk Premium

Figure 4: FFF and HP Risk Premia

Panel A: UK

Panel B: Canada
Verifying Fama’s Explanation of UIP Puzzle

1. We run the two auxiliary regressions using the HP and FFF estimates of risk premium as the dependent variables:

\[
\hat{\theta}_t^{\text{HP}} = \phi_{10} + \phi_{11} ID_t + \eta_{1t} \quad (10)
\]

\[
\hat{\theta}_t^{\text{FFF}} = \phi_{20} + \phi_{21} ID_t + \eta_{2t} \quad (11)
\]

where \( \phi_{11} = \frac{\text{cov}(\hat{\theta}_t^{\text{HP}}, ID_t)}{\text{var}(ID_t)} \) and \( \phi_{21} = \frac{\text{cov}(\hat{\theta}_t^{\text{FFF}}, ID_t)}{\text{var}(ID_t)} \). It follows that these two slope coefficients are negative if the covariances in the numerators are negative; while they are less than \(-1\) if the covariances are less than the negative variances.
Verifying Fama’s Explanation of UIP Puzzle

Figure 7: Verifying Proposition 1

Panel A: UK

Panel B: Canada
Verifying Fama’s Explanation of UIP Puzzle

1. Figure 7 plots $\hat{\phi}_{11}$ (marked by square) and $\hat{\phi}_{21}$ (marked by diamond) from regressions (10) and (11) with the same rolling windows used for Figure 1.

2. We find that majority of $\hat{\phi}_{11}$ and $\hat{\phi}_{21}$ are negative, supporting (P1).

3. Nevertheless, $\hat{\phi}_{11}$ and $\hat{\phi}_{21}$ are not necessarily less than -1, so the evidence for (P2) is elusive.
Conclusion

1. This paper advocates two nonparametric approaches of estimating the risk premium.

2. The first estimate from the Hodrick-Prescott filter is built upon the smoothness assumption motivated by consumption smoothing, sticky prices and central limit theorem in the intertemporal asset pricing framework.

3. The second estimate is more flexible, free of typical assumptions about the structural form, and entails the Fourier series approximation to underlying risk structure.

4. The covariance between the interest differential and estimated risk premium is found to be negative, which explains why the Fama coefficient is usually less than unity. However, there is mixed finding about the covariance being less than negative variance of interest differential, which is needed to explain why the Fama coefficient is sometimes negative.