Practice Exam 2, Econ 420Y (key and discussion are in the end)

Total points are 100. But it counts 25% toward your final grade. So the effective total points are 25

Note: please write legibly

Last Name First Name

We use following codes to find the optimal portfolio with three risky assets. Please answer Q1, 2, 3, 4, 5

```
library(quadprog)
miu1 = 5;s1 = 4; miu2=8; s2 = 5; miu3=7; s3 = 4.5
rho12 = -0.5; rho13 = -0.3; rho23 = 0.4
s12=rho12*s1*s2; s13=rho13*s1*s3; s23=rho23*s2*s3
miuv = c(miu1, miu2, miu3)
sigma = matrix(c(s1^2,s12,s13,s12,s2^2, s23, s13,s23,s3^2),nrow=3,ncol=3,byrow=T)
targmiu = 7

Dmat = 2*sigma
dvec = c(0,0,0)
Amat = matrix(c(1,1,1,miu1,miu2,miu3),nrow=3,ncol=2)
bvec = c(1, targmiu)
ou = solve.QP(Dmat,dvec,Amat,bvec,meq=2)
cat("Optimal weight is ", ou$solution, "\n")
cat("Min-standard deviation is ", sqrt(ou$value), "\n")
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Q1 (5 points) Please find σ_{12}

Q2 (5 points) What does the sign of σ_{12} tell us?

Q3 (5 points) Can you guess the standard deviation of the optimal portfolio is greater or less than the standard deviation of the third asset? Why?

Q4 (5 points) Please explain meq=2 in the solve.QP function

Q5 (10 points) Suppose ρ_{12} changes to 0.5. Please discuss how this change will affect the minimum variance set

Q6 (10 points) Please draw a diagram and explain the Tobin's separation theorem

Q7 (5 points) What is Capital Market Line?

Q8 (5 points) What is the weight of an asset in the Market Portfolio

Q9 (5 points) (True or False, and Why) According to CAPM, the risk of an asset stems from its variance

Q10(5 points) The risk-free rate is 3%, the expected return of market portfolio is 9%, and the stock's beta is 1.2. Find the expected return on the stock according to the CAPM

Q11 (10 points) Continue Q10. Suppose the actual return of that stock is 8%. Compute the alpha for that stock, and label that stock in the SML diagram

Q12 (10 points) Please explain the three factors used by the Fama-French Factor model

Q13 (5 points) Consider an AR(2) model

$$y_t = 0.7y_{t-1} - 0.3y_{t-2} + e_t$$

Suppose the last two observations are $y_{T-1} = 80, y_T = 100$. Please predict y_{T+1}

Q14 (5 points) Please describe how to test whether the AR(2) model is adequate

Q15 (5 points) What is the advantage of AR model over MA model?

Q16 (5 points) What is unit root test?

Brief Key and Discussion

- Q1: $\sigma_{12} = \rho_{12}\sigma_1\sigma_2 = -0.5(4)(5) = -10$
- Q2: Returns of stock 1 and stock 2 are negatively correlated (moving in opposite direction)
- Q3: Less than, since diversification leads to less risk of the portfolio relative to the individual asset that has the same expected return
- Q4: The two constraints imposed by Amat are in forms of being equal, not being greater than or equal
- Q5: The new minimum variance set is less bulging to left than the old one due to the positive correlation
- Q6: Tobin's Separation Theorem states that portfolio selection can be divided into two separate decisions: first, all investors, regardless of their individual risk preferences, will hold the same optimal portfolio of risky assets called the market portfolio; second, investors then adjust their personal risk exposure by deciding how much to invest in the risk-free asset (like Treasury bills) versus the market portfolio. Risk-averse investors lend (hold more of the risk-free asset), while risk-tolerant investors borrow (take a leveraged position in the market portfolio). In short, all investors only need two funds: a risk-free asset and a mutual fund (the market portfolio) of risky assets
- Q7: CML is the tangent straight line that connects the market portfolio and risk free asset in the sigma-mu diagram. It shows all possible combinations of a risk-free asset and the market portfolio that an investor can hold. CML illustrates the risk-return trade-off. Mathematically, the line indicates that the return of an efficient portfolio (combinations of the risk-free asset and the market portfolio) is

  $$\mu_p = \mu_f + \frac{\mu_M - \mu_f}{\sigma_M}\sigma_p$$

  where the slope of CML $\frac{\mu_M - \mu_f}{\sigma_M}$ is the Sharpe ratio of the market portfolio
- Q8: Weight of an asset in the market portfolio is the proportion of the total market value of that asset relative to the total market value of all risky assets in the economy. As a result, the market portfolio is a value-weighted portfolio, and Apple and Microsoft have much higher weights than smaller firms because of their much larger market values

- Q9: False. CAPM implies that the risk of an assets stems from its covariance with the market portfolio. Only systematic risk is compensated, and systematic risk is measured by beta, which has covariance in the numerator
- Q10: See HW5
- Q11: See HW5
- Q12: The first factor is Market Risk Premium $\mu_M - \mu_f$, which represents systematic market risk or the excess return of the market portfolio over the risk-free rate; the second factor is Size Factor - SMB ("Small Minus Big"), which measures the size effect: historically, small-cap stocks have outperformed large-cap stocks on a risk-adjusted basis (a positive loading means the stock behaves more like a small-cap company. a negative loading means it behaves more like a large-cap firm); the third factor is Value Factor - HML ("High Minus Low"), which captures the value effect: value stocks (high book-to-market ratios) tend to outperform growth stocks (low book-to-market ratios). A positive loading means the stock behaves like a value stock, while a negative loading means it behaves like a growth stock. Fama and French observed that small firms and value firms tend to have higher average returns, which can be seen as compensation for additional risk not captured by CAPM's single market factor.
- Q13: $0.7(100) - 0.3(80) = 46$
- Q14: Apply Ljung-Box test to the residual. The model is adequate if the p value is greater than 0.05 (the null hypothesis that residual is white noise cannot be rejected)
- Q15: AR model can capture long memory. Also AR model is easier to fit with OLS than MA model, which requires maximum likelihood method
- Q16: Unit root test is about the hypothesis that there is unit root in the time series, or the time series is nonstationary. The testing statistics follows a special Dickey-Fuller distribution. P value less than 0.05 leads to rejection. Unit root test is needed to determine whether it is necessary to take difference of the time series before applying the ARMA model