Practice Exam, Econ 420Y (key and discussion are in the end)

Total points are 100. But it counts 25% toward your final grade. So the effective total points are 25

Note: please write legibly

`Last Name` `First Name`

We use R to create a matrix

$$A = \begin{pmatrix} 2 & 5 & 7 \\ 3 & 1 & 4 \end{pmatrix}$$

Please answer Q1, 2, 3, 4

Q1 (10 points) Please complete the blank in the following command

```
A = matrix(c(2,3,5,1,7,4), nrow= blank A, byrow = blank B)
```

Answer: blank A =
Answer: blank B =

Q2 (5 points) What is the R result when we type command

```
A[2,3]
```

Answer:

Q3 (5 points) What is the R result when we type command

```
A[,1]*A[,2]
```

Answer:

Q4 (5 points) What is the R result when we type command

```
(A[2,]>3)+1
```

Answer:

Consider using R to simulate the return of a Casino game 100000 times: you pay two dollar to roll two dices. The casino pays you ten dollars if you see two identical numbers on the two dices, and pays you one dollar otherwise. Please answer Q5, 6, 7

The incomplete R codes are below, and please complete the blanks

```
set.seed(12345)
i=1
return = blank A
blank B
 dice1 = sample(1:6,1)
 dice2 = sample(1:6,1)
 blank C
 i = i+1
}
mean(return)
var(return)
```

Q5 (5 points) Answer: blank A =

Q6 (5 points) Answer: blank B =

Q7 (10 points) Answer: blank C =

Q8 (10 points) Please discuss fully how the variance is related to the attitude toward risk

Q9 (5 points) Please explain the Newton Method. What is the limitation of this method?

We want to apply the Bisection method to solve the equation

$$x^3 + 2x = 10$$

The incomplete R codes are below, and please complete the blanks

```
d = function(x)
{return(Blank A)}

i = 0; smallx = 0; bigx = 3
while (i<20) {
 root = Blank B
 if (d(root)*d(bigx)>0)
 {Blank C}
 else
 {Blank D}
 cat("--------------------------", "\n")
 cat("Root is ", root, "\n")
 i = i+1
}
```

Q10 (5 points) Answer: blank A =

Q11(5 points) Answer: blank B =

Q12 (5 points) Answer: blank C =

Q13 (5 points) Answer: blank D =

Q14 (5 points) Please explain what would happen if try smallx = 0; bigx = 1 in the codes above

Consider the following R codes for a quadratic programming problem

```
Dmat = matrix(c(2,0,0,4),2,2,byrow=T)
dvec = c(4,8)
Amat = matrix(c(0,0,0,0),2,2,byrow=T)
bvec = c(0,0)
solve.QP(Dmat,dvec,Amat,bvec)
```

Q15 (5 points) Please use math (either by completing the squares or taking derivatives) to find the solution x_1^* and x_2^*. How much is the minimized value?

Q16 (5 points) Continue Q1. Now consider adding the constraint using the R codes

```
Amat = matrix(c(1,-1,0,1),2,2,byrow=T)
bvec = c(5,3)
```

please mathematically show me the constraint.

Q17 (5 points) Continue Q2. Do you expect to see new solution x_1^* and x_2^* after these constraints are added? Why? (you do not need to find the new solution)

Brief Key and Discussion

- Q1: blank A = 2 since the A matrix has two rows; blank B = F since we organize the numbers by column. You cannot use lowercase f since R is case-sensitive.
- Q2: Answer is 4, which is the entity at the second row and third column. In R the bracket is used for indexing. For a matrix, the first number refers to row, and the second number refers to column.
- Q3: Answer is 10 3. The element-by-element product of first and second columns
- Q4: Answer is 1 1 2 since the parentheses evaluate whether the second row is greater than 3, and we get FALSE, FALSE, TRUE, which are stored as numbers 0, 0, 1 in computer. Then we add number 1 to each number.
- Q5: blank A is rep(-1,100000)
- Q6: blank B is $while(i <= 100000)$ {
- Q7: blank C is $if(dice1 == dice2)\ return[i] = 8$
- Q8: see page 11 of lecture notes and my discussion about how utility function being concave, linear, or convex is related to interpreting rising variance
- Q9: The New Method is a numerical method for optimizing a function, and it computes the step size as $-\frac{f'}{f''}$, which is the negative ratio of gradient (first order derivative) to Hessian (second order derivative). The Newton method fails when the Hessian is zero, or does not exist.
- Q10: blank A is

  ```
  x^3+2*x-10
  ```

  which is the distance function
- Q11: blank B is

  ```
  (smallx+bigx)/2
  ```

- Q12: blank C is

```
bigx = root
```

- Q13: blank D is

```
smallx = root
```

- Q14: The algorithm does not return the true root since the interval of smallx and bigx does not contain the root, or smallx and bigx have the same sign of d function value (i.e., $d(smallx) < 0, d(bigx) < 0$)

- Q15: The objective function is $x_1^2 + 2x_2^2 - 4x_1 - 8x_2$. After completing the squares we get $(x_1 - 2)^2 + 2(x_2 - 2)^2 - 12 \geq -12$. Thus the solutions are $x_1^* = 2, x_2^* = 2$, and the minimized value is -12. Alternatively, we can take two derivatives, set them to zero, and the first order conditions are $2x_1^* - 4 = 0, 4x_2^* - 8 = 0$. Solving them leads to $x_1^* = 2, x_2^* = 2$. Plugging those values back to $x_1^2 + 2x_2^2 - 4x_1 - 8x_2$, we get -12.

- Q16: The constraints are

$$\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}' \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \geq \begin{pmatrix} 5 \\ 3 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \geq \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

  Applying the transpose of a matrix and definition of inner product, we get

$$x_1 \geq 5, x_2 - x_1 \geq 3.$$

- Q17: Because $x_1^* = 2, x_2^* = 2$ satisfy neither of those two inequalities, so we expect to see new solutions to differ from old solutions. In other words, the two constraints are both binding. (Discussion, not required by exam) After we run those codes in R we get

```
$‘solution‘
[1] 5 8
$value
```

```
[1] 69
```

So the new solutions with constraints are $x_1^* = 5, x_2^* = 8$. The new minimized value is 69.