Take Away: Function Form

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1. Car makers are interested in how fuel efficiency and weight of vehicle are related. In particular, whether the relationship is linear or nonlinear.

2. The mtcars data are available in R. The y variable is mpg (miles per gallon), and x variable is wt (weight). The first two cars in the data are shown below

```
> data(mtcars)
> mtcars[1:2,]
               mpg cyl disp  hp drat    wt  qsec vs am gear carb
Mazda RX4     21.0   6  160 110 3.90 2.620 16.46  0  1    4    4
Mazda RX4 Wag 21.0   6  160 110 3.90 2.875 17.02  0  1    4    4
```

3. A simple regression indicates a negative relationship or opposite co-movement

```
> summary(lm(mpg~wt))$coef
             Estimate Std. Error   t value     Pr(>|t|)
(Intercept) 37.285126   1.877627 19.857575 8.241799e-19
wt          -5.344472   0.559101 -9.559044 1.293959e-10
> 5.344472/mean(mpg)
[1] 0.2660182
```

   Increasing wt by one unit is associated with average mpg going down by 5.344472. To put that number into perspective, we divide it by the average mpg and get 0.2660182. So a better interpretation is—increasing wt by one unit is associated with average mpg going down by 26 percent. We get a similar percentage change (27.17 percent) by running the log-level model:

```
> summary(lm(log(mpg)~wt))$coef
              Estimate Std. Error   t value     Pr(>|t|)
(Intercept)  3.8319136 0.08395611  45.64186 2.788963e-29
wt          -0.2717847 0.02499961 -10.87156 6.309570e-12
```

4. The results of a quadratic model with squared term is

```
> wtsq = wt^2
> summary(lm(mpg~wt+wtsq))$coef
```

```
              Estimate Std. Error   t value     Pr(>|t|)
(Intercept)  49.930811  4.2112880 11.856423 1.213142e-12
wt          -13.380337  2.5140052 -5.322319 1.035750e-05
wtsq          1.171087  0.3594456  3.258037 2.859895e-03
> turningpoint = -(-13.380337)/(2*1.171087)
> turningpoint
[1] 5.712785
```

The squared wt has a positive and statistically significant coefficient, see red and blue numbers above. Thus, the parabola faces upward, and the curve becomes flatter and flatter before the turning point, implying decreasing marginal effect of wt on mpg. mpg reaches its minimum or turning point when wt equals 5.712785. In short, a heavier car tends to be less fuel efficient. We find evidence that the relationship is nonlinear—as wt rises, the mpg falls at decreasing (not constant) rate

5. We also find evidence of interaction—how wt affects mpg depends on horsepower (hp)

```
> inte = wt*hp
> summary(lm(mpg~wt+wtsq+inte))$coef
                Estimate Std. Error   t value     Pr(>|t|)
(Intercept)  47.53476142 3.79562675 12.523561 5.403651e-13
wt          -11.77192966 2.27867260 -5.166135 1.761057e-05
wtsq          1.26659430 0.31849311  3.976834 4.471198e-04
inte         -0.00749064 0.00245655 -3.049252 4.971184e-03
```

A heavier car with more horsepower is even more fuel inefficient

6. Residuals can be used to evaluate each car's performance in terms of fuel efficiency. For instance, the first two cars (Mazda RX4, Mazda RX4 Wag) underperform as their residuals are negative (actual mpg is less than predicted mpg). The best-performing car is Fiat 128

```
> resid(lm(mpg~wt+wtsq+inte))[1:2]
 -2.2279132 -0.7907423
> mtcars[which.max(resid(lm(mpg~wt+wtsq+inte))),]
           mpg cyl disp hp drat  wt  qsec vs am gear carb
 Fiat 128 32.4   4 78.7 66 4.08 2.2 19.47  1  1    4    1
```