Eco311 Optional Reading: Fixed Effect Panel Data Model

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1. We obtain **panel data** by *repeatedly* observing entities such as states, counties, and households. For instance, consider the MURDER data in the Wooldridge package

   ```
   > library(wooldridge)
   > data(murder)
   > head(murder[,1:6])
     id state year mrdrte exec unem
   1  1    AL   87    9.3    2  7.8
   2  1    AL   90   11.6    5  6.8
   3  1    AL   93   11.6    2  7.5
   4  2    AK   87   10.1    0 10.8
   5  2    AK   90    7.5    0  6.9
   6  2    AK   93    9.0    0  7.6
   > attach(murder)
   ```

   For each state, we observe three times murder rate (mrdrte) and past execution of convicted murderers (exec), so this is panel data.

2. For panel data we need two (or even more) subscripts to distinguish observations. For instance $mrdrte_{AL,87} = 9.3$ represents Alabama's murder rate in 1987. By contrast, we only need one subscript for time invariant variable $state_i$ or state-invariant variable $year_t$.

3. Our goal is examining whether past execution has a deterrent effect on murder. The baseline model is

   $$mrdrte_{i,t} = \beta_0 + \beta_1 exec_{i,t} + error, \quad (i = AL, AK, ..., t = 87, 90, 93) \tag{1}$$

   and R results are

   ```
   > summary(lm(mrdrte~exec))$coef
                Estimate Std. Error  t value     Pr(>|t|)
   (Intercept) 7.7657713  0.7799683 9.956521 2.880226e-18
   exec        0.2480691  0.1962792 1.263859 2.082284e-01
   ```

```
> summary(lm(mrdrte~exec))$r.squared
[1] 0.01046767
```

There are two red flags. First, the coefficient of exec 0.2480691 is positive, meaning that past execution and murder rate are positively correlated. This finding is counter-intuitiveIf if we believe in the deterrent effect (implied by a negative β_1). Second, R-squared 0.01046767 is near zero. So model (1) may suffer from substantial omitted variable bias (i.e., 0.2480691 is very likely to be biased).

4. Since we observe each state multiple times, we may explain variation of murder rate by just using state dummy variables

```
> summary(lm(mrdrte~state))$coef
               Estimate Std. Error     t value     Pr(>|t|)
(Intercept)  8.8666668   2.070222  4.28295559 4.181179e-05
stateAL      1.9666669   2.927735  0.67173656 5.032700e-01
stateAR      0.4999998   2.927735  0.17078041 8.647347e-01
stateAZ     -0.9333334   2.927735 -0.31879022 7.505375e-01
stateCA      3.0000000   2.927735  1.02468279 3.079369e-01
stateCO     -3.6000001   2.927735 -1.22961936 2.216685e-01
stateCT     -3.4333334   2.927735 -1.17269254 2.436506e-01
stateDC     55.3000011   2.927735 18.88831974 4.439385e-35
...
stateWY     -5.4333334   2.927735 -1.85581440 6.636707e-02
> mean(mrdrte[state=="AK"])
[1] 8.866667
> mean(mrdrte[state=="AL"])-mean(mrdrte[state=="AK"])
[1] 1.966667
> summary(lm(mrdrte~state))$r.squared
[1] 0.8979038
```

(a) The variable state is a **string**, and R *automatically* treats it as a **factor**. That means R generates *a group of dummy variables*, one for each state. The regression behind the lm function looks like

$$mrdrte_{i,t} = \alpha_0 + \alpha_1 D_{AL} + \alpha_2 D_{AR} + ... \tag{2}$$

where the dummy variable $D_{AL} = 1$ for Alabama, and 0 otherwise.

(b) R drops the dummy variable for Alaska in order to avoid **dummy variable trap**. Thus, Alaska is the base group, and it is captured by the intercept

$$8.8666668 = E(mrdrte|state = AK)$$

(c) The coefficient of Alabama dummy measures the difference between AL and AK

$$1.9666669 = E(mrdrte|state = AL) - E(mrdrte|state = AK)$$

(d) We see that there is a significant difference between DC and AK. The t value for DC dummy coefficient is 18.88831974 (greater than 1.96), rejecting the null hypothesis of equal means between DC and AK.

(e) Most importantly, the R-squared 0.8979038 is near 1, implying that state dummy variables can explain a big portion of variation in murder rate.

5. The previous finding motivates a **dummy variable model** (DVM) that takes into account the state effect

$$mrdrte_{i,t} = \beta_0 + \beta_1 exec_{i,t} + \sum_{i \neq AK} \alpha_i D_i + error, \tag{3}$$

where D_i equals one for the i-th state, and zero otherwise. Its coefficient is α_i.

6. Intuitively, model (3) is better than (1) since it includes control variables (state dummy variables) in addition to the key regressor exec. We expect the results from model (3) are closer to the truth (less biased) than model (1).

7. Let's look at $\sum_{i \neq AK} \alpha_i D_i$ more closely. We can show

$$\alpha_{AL} D_{AL} + \alpha_{AR} D_{AR} + ... = \alpha_{AL} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix} + \alpha_{AR} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ \vdots \end{pmatrix} + ... = \begin{pmatrix} \alpha_{AL} \\ \alpha_{AL} \\ \alpha_{AL} \\ \alpha_{AR} \\ \alpha_{AR} \\ \alpha_{AR} \\ \vdots \end{pmatrix} \tag{4}$$

That is why the DVM (3) can be *equivalently* written as

$$mrdrte_{i,t} = \beta_0 + \beta_1 exec_{i,t} + \alpha_i + error, \quad \texttt{(one way FE model)} \tag{5}$$

where α_i is called **state fixed effect**, and model (5) is called one way fixed effect (FE) model. Note that the state fixed effects are actually coefficients of state dummy variables.

8. The estimation results of DVM (3) or one way FE model (5) is

```
> summary(lm(mrdrte~exec+state))$coef
               Estimate Std. Error     t value     Pr(>|t|)
(Intercept)  8.86666679  2.0767862  4.26941718 4.433939e-05
exec        -0.10610750  0.1777927 -0.59680466 5.519733e-01
stateAL      2.28498935  2.9850585  0.76547557 4.457735e-01
...
stateWY     -5.39796423  2.9376171 -1.83753159 6.907131e-02
> summary(lm(mrdrte~exec+state))$r.squared
[1] 0.8982626
```

We make big progress by including the state dummy variables (or controlling for state fixed effect) in regression. Now the coefficient of exec -0.10610750 is negative, consistent with the deterrent hypothesis. Nevertheless, its t value -0.59680466 indicates that the correlation between exec and mrdrte is not significant.

9. Next we try to run a **two-way FE model** that augments the one-way FE model with *dummy variables for years*

$$mrdrte_{i,t} = \beta_0 + \beta_1 exec_{i,t} + \alpha_i + \gamma_t + error, \quad \texttt{(two way FE model)} \tag{6}$$

where γ_t represents coefficients of a group of dummy variables, one for each year (called year fixed effect). The results are

```
> summary(lm(mrdrte~exec+state+factor(year)))$coef
                  Estimate Std. Error     t value     Pr(>|t|)
(Intercept)     7.83505679  2.0679488  3.78880596 2.600349e-04
exec           -0.12726707  0.1759911 -0.72314479 4.712960e-01
```

```
stateAL         2.34846806  2.9169794  0.80510273 4.226898e-01
stateAR         0.66968926  2.8783807  0.23266181 8.165044e-01
...
stateWY        -5.39091104  2.8693996 -1.87875923 6.321934e-02
factor(year)90  1.36310314  0.6974614  1.95437794 5.347759e-02
factor(year)93  1.73172688  0.6988685  2.47790081 1.490845e-02
> summary(lm(mrdrte~exec+state+factor(year)))$r.squared
[1] 0.904856
```

More progress has been made—the absolute t value of exec gets bigger $0.72314479 > 0.59680466$, though it is still less than 1.96. The t value of 1993 dummy variable 2.47790081 exceeds 1.96, implying that there is a significant difference between 93 and 87.

10. Pay attention that we should treat year as a **categorical** variable as opposed to a numeric variable. So we have to use **factor** function to *coerce* it into a factor.

11. A key question is, what really is the state fixed effect? Let's decompose α_{AL} further as

$$\begin{pmatrix} \alpha_{AL} \\ \alpha_{AL} \\ \alpha_{AL} \end{pmatrix} = \begin{pmatrix} \texttt{in south} \\ \texttt{in south} \\ \texttt{in south} \end{pmatrix} + \begin{pmatrix} \texttt{execution is allowed} \\ \texttt{execution is allowed} \\ \texttt{execution is allowed} \end{pmatrix} + ... \tag{7}$$

The point is, because α_{AL} is time-invariant, it *approximates* or captures all **time-invariant unobserved factors** such as location (AL being in south), legal system (death penalty being allowed in AL), and so on.

12. In a similar way, we can show that the year fixed effect capture all state-invariant unobserved factors. For instance, if every state sees a common trend of rising gun ownership, then γ_t captures that national trend. More generally, we can modify the model to allow for **state-specific trend**

$$mrdrte_{i,t} = \beta_0+\beta_1 exec_{i,t}+\alpha_i+\theta_i t+error, \quad (\texttt{FE model with state-specific trend}) \tag{8}$$

13. **By including the state fixed effect and year fixed effect, we are able to control for time-invariant and state-invariant omitted variables** (such as legal

system and trend in gun ownership). Therefore, **we can get less biased estimate of causal effect by running fixed effect panel data models**.

14. Watch out for a caveat—to use FE model, the key regressor cannot be time-invariant. Otherwise, the regressor can be written as a linear combination of those state dummy variables. In other words, there would be a *perfect multicollinearity* if the key regressor does not vary over time. For instance, an FE model cannot disentangle the effect of being in AL from being in South. Thus the FE model cannot be used to estimate the effect of being south on the murder rate.

15. There is another way to understand why panel data can help mitigate omitted variable bias. This time, we view α_i in (5) as time-invariant unobserved factor (other than a set of dummy variables). Bias will arise if there is endogeneity $cov(\alpha_i, x_{it}) \neq 0$. So our goal is to remove α_i using the so-called **within** or **demean** transformation

$$mrdrte_{i,t} - \overline{mrdrte}_i = \beta_1(exec_{i,t} - \overline{exec}_i) + error, \quad (\texttt{within model}) \tag{9}$$

where $\overline{x}_i \equiv \frac{\sum_{t=1}^T x_{it}}{T}$ denotes average over time for given i. Since α_i remains constant over time, $\alpha_i - \overline{\alpha}_i = 0$. To sum up, subtracting entity-specific time average eliminates troublemaker α_i. R codes below undertake the within transformation via a user-defined function and obtain the same coefficient estimate -0.1061075 for exec as model (3)

```
> p.demean = function(idv, yv) {
+ yb = rep(0, length(yv))
+ for (i in unique(idv)) yb[idv==i] = yv[idv==i]-mean(yv[idv==i])
+ return(yb)
+ }
> summary(lm(p.demean(id, mrdrte)~p.demean(id, exec)-1))$coef
                     Estimate Std. Error    t value  Pr(>|t|)
p.demean(id, exec) -0.1061075  0.1449282 -0.7321386 0.4652113
```

16. To summarize, **fixed effect panel data models are suitable for causal study because they reduce bias by controlling for time invariant and state invariant unobserved omitted variables**.