

Eco311 Optional Reading: Examine Economics Models

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1. This note provides an example that links eco311 to eco315. The goal is to show how to run regressions implied by a particular eco model, and how to test hypotheses imposed by the eco model.
2. Consider an optimization problem of maximizing Cobb-Douglas utility subject to a budget constraint

$$\max_{x,y} x^a y^{1-a}, \quad s.t. \quad p_x x + p_y y = w \quad (1)$$

We can set up a Lagrange to solve this problem

$$L(x, y, \lambda) = x^a y^{1-a} - \lambda(p_x x + p_y y - w) \quad (2)$$

The first order conditions are

$$\frac{\partial L}{\partial x} = 0 \Rightarrow a x^{a-1} y^{1-a} - \lambda p_x = 0 \quad (3)$$

$$\frac{\partial L}{\partial y} = 0 \Rightarrow (1-a) x^a y^{-a} - \lambda p_y = 0 \quad (4)$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow p_x x + p_y y - w = 0 \quad (5)$$

Equation (3) dividing by (4) leads to

$$\frac{a}{1-a} \frac{y}{x} = \frac{p_x}{p_y} \quad (6)$$

Combining (5) and (6) leads to the final solutions

$$x^* = \frac{aw}{p_x} \quad (7)$$

$$y^* = \frac{(1-a)w}{p_y} \quad (8)$$

3. Taking natural log of (7) suggests a *restricted* regression

$$\log x = \log a + \log w - \log p_x \quad (9)$$

Accordingly, we can run an *unrestricted* regression

$$\log x = \beta_0 + \beta_1 \log w + \beta_2 \log p_x + error \quad (10)$$

Then test the *joint hypothesis* imposed by this particular model

$$H_0 : \beta_1 = 1, \beta_2 = -1 \quad (11)$$

We can conduct an F or Wald test.

4. Suppose we also have data for p_y , then the unrestricted model and joint hypothesis become

$$\log x = \beta_0 + \beta_1 \log w + \beta_2 \log p_x + \beta_3 \log p_y + error \quad (12)$$

$$H_0 : \beta_1 = 1, \beta_2 = -1, \beta_3 = 0 \quad (13)$$

5. The ratio of optimal bundle is

$$\frac{x^*}{y^*} = \frac{a}{1-a} \frac{p_y}{p_x} \quad (14)$$

If we also get data for y , we may run another unrestricted regression

$$\log(x/y) = \beta_0 + \beta_1 \log(p_y/p_x) + \beta_2 \log(w) + error \quad (15)$$

and test

$$H_0 : \beta_1 = 1, \beta_2 = 0 \quad (16)$$

6. Finally, we may run a Seemingly Unrelated Regression (SUR)—a system of two regressions, one for $\log x$ and the other for $\log y$

$$\begin{cases} \log x = \beta_0 + \beta_1 \log w + \beta_2 \log p_x + error_x \\ \log y = \alpha_0 + \alpha_1 \log w + \alpha_2 \log p_y + error_y \end{cases} \quad (17)$$

and test cross-equation restriction $H_0 : \beta_1 = \alpha_1 = 1, \beta_2 = \alpha_2 = -1$

7. For teaching purpose, this note makes a lot of simplifications. For instance, we ignore the endogeneity issue that quantity and price may be simultaneously determined. But the key message is clear: eco model should have some testable hypotheses, and eco 311 teaches you the tool to test those hypotheses and verify the eco theory.