\[
\hat{\beta}_{2SLS} = \frac{\sum y_i \hat{x}_i}{\sum \hat{x}_i^2} = \frac{\sum y_i \hat{x}_i}{\sum \hat{x}_i^2}
\]

First stage: \( X = c_1z_1 + c_2z_2 + \cdots + c_mz_m + \text{other exogenous regressors} + v \)

Keep fitted value \( \hat{X} \), and residual \( \hat{v} \)

Second stage, use \( \hat{X} \) as IV

\[
\hat{\beta}_{2SLS} = \frac{\sum y_i \hat{x}_i}{\sum \hat{x}_i^2}
\]

Three tests

1. \( H_0: \text{cov}(X, u) = 0 \) (X is exogenous): Hausman Test
   \( Y = X\beta + \epsilon \) and test \( d = 0 \); small \( p \) \( \Rightarrow \) 2SLS is needed

2. \( H_0: \text{cov}(z, X) = 0 \) (Z is weak IV): Stock-Yogo Test
   Test \( C_1 = c_2 = \cdots = c_m = 0 \) in first stage regression
   \( F \text{ test} > 10 \Rightarrow Z \) is not weak IV

3. \( H_0: \text{cov}(z, u) = 0 \) (Z is exogenous)
   \( \Rightarrow \) if \( \# \text{ of } z = \# \text{ of endogenous regressors}, \) untestable, model is just-identified
   \( \Rightarrow U = a_1z_1 + \cdots + a_mz_m + \text{other exogenous regressor} \quad nR^2 \sim x_q^2 \)