Exam 1, Econ 311, Spring 2014

Total points are 100. But it counts 20% toward your final grade. So the effective total points are 20

Note: (i) show me your work in details in order to get partial credits; (ii) round your answer to 2 decimal spaces

Last Name First Name

You may use the following facts to answer some questions. Let $Z \sim N(0, 1)$ then

$$P(Z < 1.96) = 0.975 \quad P(Z < 1.645) = 0.950 \quad P(Z < 1.28) = 0.900$$

Q1, 2, 3, 4, 5, 6 and 7 are based on Table 1, which summarizes the random variable $y$:

```
. sum y, detail
```

Table 1

<table>
<thead>
<tr>
<th>Percentiles</th>
<th>Percentile Value</th>
<th>Largest Value</th>
<th>Percentile Value</th>
<th>Largest Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>.5436599</td>
<td>.2617587</td>
<td>5%</td>
<td>1.371783</td>
</tr>
<tr>
<td>5%</td>
<td>1.371783</td>
<td>.2717461</td>
<td>10%</td>
<td>1.607834</td>
</tr>
<tr>
<td>25%</td>
<td>2.298603</td>
<td>.9708371</td>
<td>50%</td>
<td>3.034506</td>
</tr>
<tr>
<td>75%</td>
<td>3.651639</td>
<td>5.11299</td>
<td>90%</td>
<td>4.266587</td>
</tr>
<tr>
<td>95%</td>
<td>4.627887</td>
<td>5.313699</td>
<td>99%</td>
<td>5.278006</td>
</tr>
</tbody>
</table>

Q1 (5 points) Find the standard error of $\bar{y}$
Q2 (5 points). Under what condition the sample mean $\bar{y}$ is an unbiased estimator of $\mu_y$?

Q3 (10 points). Is the distribution of $y$ symmetric? If not, skewed to left or right? What does that skewness mean?

Q4 (10 points). Find the two tailed t test for the null hypothesis $H_0 : \mu_y = 2$ against $H_1 : \mu_y \neq 2$, and draw a conclusion (using the critical value of 1.96)

Q5 (5 points). Find the 90% confidence interval for $\mu_y$, and interpret the confidence interval.

Q6 (5 points). Find the 90% confidence interval for $\mu_y$ but assuming the sample size is 50, other than 200. All other numbers in Table 1 remain unchanged.
Q7 (5 points). Suppose $y$ follows a normal distribution. Find the value $x$ so that with 95% probability $y$ will take a value greater than $x$.

Q8 (5 points). What does i.i.d sample mean? Please explicitly write down the assumptions for the i.i.d sample.

Q9 (5 points). In practice how can we obtain a random sample if the population is the distribution of family incomes of all Miami students? Be specific!

Q10 (5 points). Consider a non i.i.d sample with three observations $\{y_1, y_2, y_3\}$, where $E(y_1) = \mu, E(y_2) = \mu, E(y_3) = \mu$ and $\text{var}(y_1) = 1, \text{var}(y_2) = 2, \text{var}(y_3) = 3$. The sample mean is $\bar{y} = \frac{y_1 + y_2 + y_3}{3}$. Please find $E(\bar{y})$.
Q11 (10 points). Continue Q10. Please find \( \text{var}(\bar{y}) \) assuming \( \text{cov}(y_i, y_j) = 0 \), for all \( i \neq j \).

Q12 (5 points). Continue Q10. Suppose we observe \( y_i' \equiv y_i + e_i \) where \( y_i \) is the true value, and \( e_i \) is the error in measurement. Is \( \bar{y}' = \frac{y_1' + y_2' + y_3'}{3} \) an unbiased estimator for \( \mu_y \) when \( E(e_1) > 0 \), \( E(e_2) > 0 \), and \( E(e_3) > 0 \)? Please explain.

Q13 (5 points). Consider a Bernoulli random variable for the decision of a voter

\[
y = \begin{cases} 
1, & \text{if voting for Republican} \\
0, & \text{if voting for Democratic}
\end{cases}
\]

and \( P(y = 1) = 0.6 \). Please find \( \text{var}(y) \)

Q14 (5 points). Continue Q13. Let \( \bar{y} = \frac{y_1 + \cdots + y_n}{n} \) be the sample proportion of voters who vote for Republican, where the sample is a random sample. Please tell me the following statement is true or false and why “\( \bar{y} \) cannot follow normal distribution even in large sample because \( y \) is a discrete random variable.”
Q15 (5 points). Let \( \bar{y} = \frac{y_1 + \ldots + y_n}{n} \) be the sample mean. Prove that \( \sum_{i=1}^{n} (y_i - \bar{y}) = 0 \)

Q16 and Q17 are based on the following result of a simple regression

```
. reg y x
```

```
Source | SS    df MS      Number of obs = 200
--------+--------------------------------------
Model   | 45.0853679 1 45.0853679    F( 1, 198) = 38.20
Residual| 233.711783 198 1.18036254   Prob > F = 0.0000
--------+--------------------------------------
Total   | 278.797151 199 1.40099071   R-squared = 0.1617
         |                                      Adj R-squared = 0.1575
         |                                      Root MSE = 1.0864

------------------------------------------------------------------------------
y | Coef.   Std. Err.     t    P>|t|     [95% Conf. Interval]
-------------+---------------------------------------------------------------
x |  .4549946  .0736201  6.18   0.000     .3098146   .6001747
_cons |  1.981414   .0769888 25.74   0.000     1.82959   2.133237
------------------------------------------------------------------------------
```
Q16 (5 points) How to interpret the coefficient of $x$, which is 0.4549946?

Q17 (5 points) Explicitly write down and explain the assumption under which the coefficient of $x$ has causal interpretation.