Handout B: The Neoclassical Consumption Model

Keynesian Theory implies that consumption is a function of current disposable income. The Keynesian consumption function and marginal propensity to consume (MPC) are

\[ C = \bar{C} + MPC \times (Y - T), \quad 0 < MPC < 1 \]

Through multiplier effect, a tax cut of one dollar can increase planned expenditure (PE) by

\[ \frac{dPE}{d(-T)} = \frac{MPC}{1 - MPC} \]

dollars. For instance let \( MPC = 0.8 \), then the tax-cut multiplier = 4. So one dollar tax-cut can increase expenditure by 4 dollars. Thus Keynesian theory implies that a tax cut is very effective in stimulating demand.

**Exercise**: the tax cut multiplier (falls rises) when MPC falls. This implies that tax cut is (more less) effective in stimulating expenditure when MPC falls.

By contrast, the neoclassical consumption model is based on microeconomic foundation by applying the consumer theory to the problem of intertemporal optimization.

From Handout A, we know that if a person lives two periods, his intertemporal optimization problem is
Now assume the intertemporal utility function $U$ takes the additive form of

$$U(C_1, C_2) = u(C_1) + \beta u(C_2)$$  \hspace{1cm} (2)

where $0 < \beta \leq 1$ measures the extent to which consumers are patient. If $\beta = 1$ then this person does not discount future consumption, so is very patient. The instantaneous utility function $u(\cdot)$ has the following two properties:

$$u' = \frac{du}{dc} > 0 \hspace{1cm} (3)$$

$$u'' = \frac{d^2u}{dc^2} = \frac{du'}{dc} < 0 \hspace{1cm} (4)$$

Property (3) implies that as consumption rises, utility _______

Property (4) implies that as consumption rises, marginal utility _

The utility curve that represents $u$ looks like

According to equation (14) in Handout A, the FOC is
\[
\frac{du/dC_1}{du/dC_2} = \beta (1 + r) \tag{5}
\]

Equation (5) is Euler equation, the core of neoclassical consumption theory.

Exercise: derive (5).

Euler equation says that at optimum this person must be indifferent between consuming one more unit in period 1 on the one hand and saving that unit and consuming it in period 2 on the other.

If he consumes today one more unit, his utility rises by __________

If he saves that unit instead, he gets to consume __________ units in the future, each giving him _____ extra units of utility. This future utility must be discounted by the weight \(\beta\).

Consider a particular instantaneous utility function. Let

\[u(c) = \log(c) \tag{6}\]

Exercise: show the log utility function (6) satisfies (3) and (4).

For this log utility function, Euler equation (5) becomes:
\[
\frac{C_2}{C_1} = \beta (1 + r) \quad (7)
\]

Consider the special case with \( \beta = 1 \). Then we can show that
\[
C_1 = \frac{1}{2} \overline{W} \quad (8)
\]
\[
C_2 = \frac{1}{2} (1 + r) \overline{W} \quad (9)
\]

where \( \overline{W} = Y_1 + \frac{Y_2}{1+r} \) denotes the present value of income, or permanent income.

**Exercise:** derive (7), (8) and (9).

Equation (8) shows that current (period 1) consumption depends on permanent income, which includes not only current income, but also future income.

Furthermore equation (8) makes it clear that what really matters is permanent income, not current income.

Intuitively permanent income matters because people are forward-looking.

Now we can generalize equation (8). Suppose this person lives \( N \) periods. Result (8) then becomes
\[ c_1 = \frac{1}{N} \bar{W} \]  

**Homework:** please obtain the marginal propensity to consume \( dc_1/dy_1 \) based on (10). Compare to the MPC in the Keynesian model. How does this difference affect the tax-cut multiplier? What is the implication on the effectiveness of tax cut on stimulating demand?

**Ricardian Equivalence**

Now we can understand a hypothesis called **Ricardian Equivalence (RE)**. RE says that consumption depends on the present value of current and future taxes, and is invariant to the timing of taxes. That is, a tax cut is ineffective in stimulating consumption because a future tax hike will follow and the present value of all taxes remains unchanged.

**Exercise:** show Ricardian Equivalence.

Step 1: modify a person’s two-period intertemporal budget constraint to account for tax

Step 2: assume government obeys the No-Ponzi-Game condition. That is, the government’s intertemporal budget constraint is

\[ T_1 + \frac{T_2}{1+r} = E_1 + \frac{E_2}{1+r} \]
where $T$ denotes tax income, and $E$ government expenditure.

Step 3: show that $T_2$ must rise after $T_1$ falls.

Step 4: show that $C_1$ remains unchanged if $E_1 + \frac{E_2}{1+r}$ remains unchanged. In other words $C_1$ depends on present value of taxes.

The Random Walk Hypothesis for Consumption

When $\beta = 1, r = 0$, Euler equation (7) implies that

$$E(C_2|C_1) = C_1$$

(11)

where $E(\quad | \quad)$ denotes the conditional mean.

Equivalently, we can write

$$C_2 = C_1 + \nu, \quad E(\nu|C_1) = 0.$$  

(12)

We can generalize (12) to more than two periods

$$C_t = C_{t-1} + \nu_t, \quad E(\nu_t|C_{t-1}) = 0.$$  

(12')

An error term that satisfies $E(\nu_t|C_{t-1}) = 0$ is called martingale difference. Because $C_t - C_{t-1} = \nu_t$, $C_t$ is called martingale, or random walk.
Equation (12’) shows that the consumption is a random walk, a particular stochastic process.

Because consumption is the biggest component of GDP, it follows that DGP is random walk as well.

A random walk has following three statistical properties:

(1) the series is highly serially correlated

(2) shock has permanent effect on the series

(3) two random walks tend to move together if they are co-integrated

These properties can explain the phenomenon that macro time series tend to move together after there is a good or bad news (shock)


**Consumption Smoothing**

The neoclassical consumption model also explains people’s preference for smooth consumption.

It is easy to see smooth consumption occurs, i.e., $C_2 = C_1$, when $\beta = 1, r = 0$ in equation (7)
Basically people prefer smooth consumption because of the decreasing marginal utility.

If $C_1 = c_{high}, C_2 = c_{low}, u^{choppy} = u(C_1) + u(C_2) =$

If $C_1 = C_2 = \frac{1}{2}(c_{high} + c_{low}), u^{smooth} = u(C_1) + u(C_2) =$

The shape of $u$ determines that $u^{smooth} > u^{choppy}$

Because people prefer smooth consumption, permanent income instead of current income matters for consumption.

**Borrowing Constraint**

One important assumption made by neoclassical consumption model is that people can borrow freely if they want to. Next we show if there is borrowing constraint, then most of conclusions we get so far in this handout become invalid.

Let’s repeat equation (8) here:

$$C_1 = \frac{1}{2} \bar{W} = \frac{1}{2} \left( Y_1 + \frac{Y_2}{1 + r} \right)$$

For simplicity, let $r = 0$. So $C_1 = \frac{1}{2}(Y_1 + Y_2)$

Assume $Y_2 > Y_1$
Step 1: it follows that $C_1 = \ldots Y_1$

Step 2: If there is no borrowing constrain, $C_1 = \ldots$

Step 3: If there is borrowing constrain, $C_1 = \ldots$

Now we see the optimal consumption (derived from microeconomic foundation) in the presence of borrowing constraint is the same as Keynesian consumption function, i.e., consumption depends on current income instead of permanent income.

As a result, borrowing constraint causes tax cut to be highly effective in stimulating demand and Ricardian Equivalence to fail.

Poor people are more likely to be subject to borrowing constraint than rich people.

Discuss Is Ricardian Equivalence applicable or not in the context of current US economy?
Handout C: The Neoclassical Labor Market Model

This handout is concerned with the Real Business Cycle theory’s explanation about fluctuation (cycle) in labor market.

Again we assume a person lives two periods. Now his intertemporal choices are working vs. leisure. Suppose each period he has $T$ time to allocate. Denote working time by $L$ (labor), so $T - L$ is the leisure time. The person tries to maximize the utility brought by leisure.

Meanwhile the person faces a tradeoff (budget constraint). If he chooses to have more leisure in one period then he must have less leisure in the other period. That is because the person has to work certain time so that the present value of current and future labor income is equal to the present value of current and future expenditure.

His **dynamic** optimization problem is

\[
\max u(T - L_1) + \beta u(T - L_2) \quad s.t \quad L_1 + \frac{wL_2}{1 + r} = \bar{E} \quad (1)
\]

where we normalize the wage in period 1 to one. $w$ denotes the relative wage in the second period. $\bar{E}$ represents the present value of current and future expenditure. The link between this problem and the problem of intertemporal choices for consumption vs. saving becomes clear if we let $L_1 = Y_1$, $wL_2 = Y_2$, $\bar{E} = C_1 + \frac{C_2}{1 + r}$.
The FOC is

\[
\frac{du/d(T - L_1)}{du/d(T - L_2)} = \beta (1 + r) \frac{1}{w} \tag{2}
\]

If we adopt the log utility function \( u = \log \), then (2) becomes

\[
\frac{T - L_2}{T - L_1} = \beta (1 + r) \frac{1}{w} = \beta c \tag{3}
\]

where \( c = \frac{1+r}{w} \).

It follows that

\[
L_1 = \frac{\bar{E}}{1 + \beta} - \frac{(1 - \beta c)T}{(1 + \beta)c} \tag{4}
\]

**Homework** derive (4)

To make sense of (4), consider how rising \( w \) affects \( L_1 \). We can show

\[
\frac{\partial L_1}{\partial w} = \frac{\partial L_1}{\partial c} \frac{\partial c}{\partial w} < 0 \tag{5}
\]

This implies rising relative wage in the second period causes the person to work less time in the first period. This result is intuitive since working in the second period can earn relatively more money.
Exercise: show that rising \( r \) causes intertemporal labor substitution as well, but in this case, \( L_1 \) rises. This result is intuitive too since rising interest rate in effect raises the wage for the first period.

**Real Business Cycle (RBC) Theory**

More generally we can extend (4) as

\[
L_1 = f(w, r, \beta, \bar{E})
\]

(6)

RBC calls the change in labor supply the *intertemporal labor substitution* (ILS). ILS can be generated by change in any parameter of the function \( f \) in (6). For example, change in people’s preference can be formulated as change in \( \beta \). Change in technology can cause marginal product of labor and \( w \) to change. Change in world loanable fund market can cause \( r \) to change.

**RBC theory believes ILS is the main reason for fluctuation in labor market.**

RBC theory uses equation (6) to explain the fluctuation in the labor market. If the number of unemployed workers rises, RBC believes it is because people choose to work less. People make new decision of supplying labor because of change in parameter of the function \( f \) in (6). In other words the unemployment is a natural adjustment process in the labor market. Government needs to do nothing.

The fluctuation in labor (input) market can spread to output market via the production function
\begin{equation}
Y = F(K, L) \quad (7)
\end{equation}

We call the fluctuation in \(Y\) business cycle. According to RBC, business cycle can be a natural outcome of adjustment in the labor market.

In graph, intertemporal labor substitution causes long run aggregate supply curve to shift. Moreover, it causes output to deviate from its average level. This is in sharp contrast of Keynesian explanation that output deviates from its average level due to shift of AD curve.

Below table gives a brief comparison of Keynesian theory and real business cycle theory.

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<th>Keynesian Theory</th>
<th>RBC Theory</th>
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<td>Sticky Price</td>
<td>Flexible Price</td>
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<tr>
<td>Key assumption 2</td>
<td>Market Failure</td>
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<td>Microeconomic Foundation</td>
<td>No</td>
<td>Yes</td>
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<tr>
<td>Causes of Business Cycle</td>
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<td>Policy Recommendation</td>
<td>Stabilization Policy that shifts AD curve</td>
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The new Keynesian theory tries to improve the old Keynesian theory by rebuilding the whole theory on a solid microeconomic foundation. Nevertheless, the new Keynesian theory stills emphasizes market failure, so believes that government policy is still needed to correct market failure.