Chapter 7: Economic Growth without Technological Progress (A Very Long Run Model)

In this chapter we understand what causes the differences in the growth in income over time and across countries.

Solow Growth Model

The Solow model shows how saving, population growth and technological progress affect the level of an economy’s output and its growth over time.

Assumption (1)
Solow model assumes a production function with constant returns to scale:

\[ zY = F(zK, zL) \]

Let \( z = 1/L \). It follows that

\[ Y/L = F(K/L, 1) \]

It can be rewritten as

\[ y = F(k, 1) \equiv f(k) \quad (1) \]

where the lowercase letter denotes quantity in per-worker terms.

The production function has the property that as capital per worker \( k \) rises, output per worker \( y \) rises, but at a decreasing rate (due to the diminishing marginal product of capital). See Figure 7-1.
Question: How to measure the marginal product for capital in Figure 7-1?

Question: what does the production function look like if the capital has increasing marginal product?

Question: what does Figure 7-1 look like if the number of workers rise?

Assumption (2): Solow model assumes a closed economy (so NX=0) without government expenditure (so G=0). We can write the per-worker version of the national income account identity as

\[ y = c + i \quad (2) \]

Question: what does \( i \) mean?

Question: can you show explicitly how to get the above equation?

Solow model also assumes the economy saves a fraction \( s \) of the income and consume a fraction \( 1 - s \) of income:

\[ c = (1 - s) y \quad (3) \]

where the saving rate \( s \) can be affected by government policies (example?).
Equations (1), (2) and (3) jointly imply

\[ i = sy = sf(k) \quad (4) \]

Exercise: Draw the investment curve (Figure 7-2)

Dynamics

Solow model is a dynamic model. We first show how capital stock changes over time. Then we show how output changes over time (since capital is the input).

The change of stock of per-worker capital is equal to the increase in new capital (investment) minus the wearing out of the existing capital (depreciation). Mathematically,

\[ \Delta k = investment - depreciation = sf(k) - \delta k \quad (5) \]

where \( \delta \) denotes the depreciation rate and \( \Delta \) denotes the change. We use equation (4) to get (5).

Equation (5) is the fundamental equation for Solow model.

Exercise: Draw Figure 7-4 (put investment curve and depreciation curve together)
The two curves (must, why?) intersect. So there exists a single (non-zero) capital stock $k^*$ at which the amount of investment equals the amount of depreciation. That is at $k^*$

$$\Delta k = 0$$

because

$$sf(k^*) - \delta k^* = 0 \quad (6)$$

See equation (5). We call $k^*$ the steady-state level of capital. Equation (6) shows that steady state depends on__________

Question: what happens to $k$ at $k^*$? Rise or fall?

Question: what happens to $y$ at $k^*$? Rise or fall?

Question: what happens to $Y$ at $k^*$, suppose $L$ rises?

Question: what happens to the standard of living at $k^*$?

The steady-state is significant for two reasons:

(I): an economy at the steady state will stay there (so no permanent growth in per-worker output).

(II): an economy not at the steady-state will go there (so the steady state is stable). That is, regardless of the level of capital with which the economy begins, it ends up with the steady-state level of capital.
Exercise: show why (II) holds.

Exercise: Show how the existing capital stock affects the growth rate? Read the case study on page 202.

The lesson is, at the lower capital stock, more capital is added by investment (due to high marginal product) than is removed by depreciation. So both capital and output grow fast.

Can you apply that lesson to China’s rapid growth? How about US’ slow growth?

Critical thinking:

Redraw Figure 7-4 assuming increasing marginal product for capital. Can you find the steady-state? Is it stable? What is the intuition?
We can use equation (6) to solve for the steady-state numerically. See the example in the textbook on page 199-202.

Figure 7-5 provides another reason for rapid growth: high saving rate.

Suppose the economy is at steady-state. After saving rate $s$ rises,

The depreciation line______________

The investment line______________
The steady-state capital ______________

The growth rate at the current capital stock ______________

In short, higher saving leads to faster growth, but only temporarily (why?).

Exercise: what are the long-run consequences of government budget deficit on saving and growth?
Exercise: empirical evidence supporting (or not) Solow Model, read case study on page 204. Keep in mind: association is not causation (an econometric issue).

Factors affecting saving rate include tax policy, retirement patterns, cultural difference, stability (expectation) and financial market.

**Golden Rule Level of Capital**

High saving leads to faster growth in long run, but lower current consumption. So there is a trade-off.

We want to know the optimal amount of capital accumulation from the standpoint of economic well-being.

At the steady-state $k^*$, the steady-state per-worker consumption is given by

$$c^* = y^* - i^* = f(k^*) - \delta k^* \quad (7)$$

Take first derivative with respect to $k^*$, it follows that $c^*$ is maximized when

$$f'(k^*) = \delta \quad (8)$$

We denote the steady-state $k^*$ that satisfies (8) as $k_{gold}$, called golden rule.
In words, at the golden rule level of capital, the marginal product of capital equals the deprecation rate.

The economy does not automatically gravitate toward the golden rule steady state. We need a particular saving rate to support it.

Two steps procedure:

Step 1: solve equation (8) to get $k_{gold}^{*}$

Step 2: solve equation (6) to get $s_{gold}$

See the numerical example on page 209.

**Population Growth**

The basic Solow model (without population growth) cannot explain sustained economic growth: eventually the economy approaches a steady state in which capital and output stay constant.

Now suppose the population and labor force grow at a rate $n$.

Intuitively, population growth reduces the accumulation of capital per worker much the way deprecation does. Depreciation reduces $k$ by wearing out the capital, whereas population growth
reduces $k$ by spreading the capital stock thinly among a larger population of workers.

Therefore equation (6) becomes:

$$ sf(k^*) - (\delta + n)k^* = 0 \quad (9) $$

Proof of (9) (optional)

In the steady state with population growth, capital per worker and output per worker are constant. However, because the number of workers is growing at a rate $n$, total capital and total output also grow at a rate $n$.

Population growth cannot explain sustained growth in the standard of living (output per worker), but it can help explain sustained growth in total output.

In Figure 7-12, we can show that a rise in the population growth rate will decrease the steady-state level of capital (the standard of living).
Read case study on page 216

Question: according to Solow model, how does China’s birth-control policy affects its standard of living in the long run?

Question: according to Solow model, how does the high growth rate of immigrants affects US’ standard of living in the long run?

Question: how to modify the Solow model to allow for sustained growth in the standard of living?

Chapter 8: Economic Growth with Technological Progress

We only cover sections 8-1 and 8-2 for chapter 8.

In order to explain persistent growth in standard of living (per-worker output), we need to introduce efficiency of labor denoted by \( E \) and rewrite the production function as

\[
Y = F(K, EL) \quad (10)
\]
The variable $E$ reflects workers’ productivity. As technology progresses, $E$ rises, meaning workers become more productive.

The product of $EL$ can be interpreted as effective number of labor. Even if the number of workers $L$ remains unchanged, but if $E$ doubles, the effective number of workers doubles, and the economy benefits from the increased production of goods and services (because workers become twice efficient or twice productive).

Now we can use the lowercase letter to stand for quantity in per effective worker term. That is, we can redefine

$$y = \frac{Y}{EL}, \quad k = \frac{K}{EL}$$

Then (after heavy math) equation (5) becomes

$$\Delta k = sf(k) - (\delta + g + n)k \quad (11)$$

At steady state $k^*$, the per-effective-worker capital and per-effective-worker output stay constant. However,

(A) The growth rate of the per-worker capital is__________

(B) The growth rate of the per-worker output is__________

(C) The growth rate of total capital is__________
The growth rate of total output is__________

The prediction of Solow model that the growth rate in (A) is the same as (B) is called balanced growth. In reality, balanced growth is observed.

Solow model also provides prediction for factor prices.

In chapter 3 we learn that the real factor price is equal to that factor’s marginal product. (see page 52).

At the steady state $k^*$, the growth rate of real wage (for labor) is__________________

At the steady state $k^*$, the growth rate of real rent (for capital) is__________________

In reality, the different growth rates in real wage and real rent are observed.

In reality, we also observe conditional convergence: different countries converge to different steady states. If different countries have the same steady states, the convergence is called absolute. Absolute convergence does not occur in reality.

For those who are big fan of econometrics, read case study on page 230.