Review of first order derivative

(Jing Li, Miami University)

1. This two-page note shows four examples of obtaining derivative of a differentiable function, and how to interpret and use derivative

2. The first order derivative of $y \equiv f(x)$ with respect to x is defined as a limit of change in y over the change in x

$$f'(x) \equiv \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \tag{1}$$

- Example 1: consider a linear function $f(x) = bx$

$$f'(x) = \lim_{\Delta x \to 0} \frac{b(x + \Delta x) - bx}{\Delta x} = b \tag{2}$$

- Example 2: consider a quadratic function $f(x) = x^2$

$$f'(x) = \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{2x\Delta x + \Delta x^2}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x) = 2x \tag{3}$$

- Example 3: consider a log function $f(x) = \log(x)$

$$\begin{aligned} f'(x) = \lim_{\Delta x \to 0} \frac{\log(x + \Delta x) - \log(x)}{\Delta x} &= \lim_{\Delta x \to 0} \frac{\log\left(\frac{x+\Delta x}{x}\right)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\log\left(1 + \frac{\Delta x}{x}\right)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{\Delta x}{x}}{\Delta x} = \frac{1}{x} \end{aligned} \tag{4}$$

  where we use the fact that $\lim_{\Delta x \to 0} \log\left(1 + \frac{\Delta x}{x}\right) = \frac{\Delta x}{x}$

- Example 4: consider an exponential function $f(x) = e^x$. Let $y = e^x$. Note

$$\log(y) = \log(e^x) = x.$$

  By taking derivative of x on both sides and applying the chain rule, we have

$$\frac{1}{y}\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{\frac{1}{y}} = y \tag{5}$$

  or

$$(e^x)' = e^x$$

3. Derivative measures the slope of $f(x)$.

    - Example 1: consider a linear function $f(x) = bx$. Since $f'(x) = b$ is constant, the function is a straight line (y changes at constant rate)
    - Example 3: consider a log function $f(x) = \log(x)$. Since $f'(x) = \frac{1}{x}$ is decreasing as x rises, the function is a curve that becomes flatter and flatter (y rises at decreasing rate).

4. We can obtain minimum or maximum by setting $f'(x) = 0$ (first order condition) because at that turning point the slope is zero.

    - Revised example 2: consider a quadratic function $f(x) = \beta_2 x^2 + \beta_1 x$. By setting $f'(x) = 2\beta_2 x + \beta_1 = 0$, we find the minimum or maximum of y is located at $x = \frac{-\beta_1}{2\beta_2}$. The second order derivative $f''(x) = 2\beta_2$ tells whether it is minimum or maximum.

5. You may read Math Refresher A of the textbook.