Effects of Filtering Data on Testing Asymmetry in Threshold Autoregressive Models

Jing Li
Miami University

July 2012

Working Paper # - 2012-04
Effects of Filtering Data on Testing Asymmetry in Threshold Autoregressive Models

Jing Li*

Department of Economics
Miami University

Abstract

Empirical researches on business cycle typically use filtering methods to obtain cyclical components in economic time series. This paper examines the effects of filtering data on the test for a linear autoregression against a threshold autoregression. Monte Carlo simulation shows that (1) filtering data in general reduces the power of the test, (2) the power is sensitive to the choice of filters and the specification of the trend and cyclical components, (3) filtering data with heterogeneous disturbances may improve the power. Empirical evidences for cyclical asymmetry are provided for the quarterly U.S. real GNP.

Keywords: Band Pass Filter, Hodrick-Prescott Filter, Moving Average Filter, Threshold Autoregression, Simulation

*Department of Economics, Farmer School of Business, Miami University, Oxford, OH 45056, USA. Phone: 001.513.529.4393, Fax: 001.513.529.6992, Email: lij14@muohio.edu.
Introduction

Many economic time series behave asymmetrically over the business cycle. Sichel (1993) defines two types of asymmetry: deepness (troughs are more pronounced than peaks) and steepness (contractions are steeper than expansions). The deepness and steepness can be captured by the threshold autoregressive (TAR) model of Tong (1983) and the momentum threshold autoregressive model (MTAR) of Enders and Granger (1998), respectively. Examples of applications of the threshold model include Cao and Tsay (1992), Galbraith (1996), Hansen (1997), Potter (1995), and Rothman (1998). Sometimes the interest is on testing asymmetry. For the threshold model, testing asymmetry amounts to testing a linear autoregression against a threshold autoregression (called the threshold test thereafter). Chan (1990) and Hansen (1996) show that the asymptotic null distribution of the threshold test is nonstandard because of the Devies problem raised by Davies (1977) and Davies (1987).

To test cyclical asymmetry, the first step is to filter the data and extract the cyclical component. For instance Neftci (1984) uses first differenced data. The first difference filter is essentially a high pass filter. The more sophisticated filters are the Hodrick-Prescott (HP) filter of Hodrick and Prescott (1997) and the Band Pass (BP) filter of Baxter and King (1999). As shown by Canova (1998) filtering or detrending data has now become a standard practice in business cycle macroeconomic research.

The aim of the present paper is to examine the effect of filtering/detrending data on the performance of the threshold test. This topic is important because the original threshold test is developed for unfiltered data, and its performance when applied to filtered data is largely unknown. One related study is Psaradakis and Sola (2003), which investigate the powers of the skewness-based test, the Markov-Chain test and the Time-Reversibility test. By focusing on the threshold test, this study can be seen as a complement to Psaradakis and Sola (2003)\textsuperscript{1}.

This paper and Psaradakis and Sola (2003) consider different mechanisms for asymmetry. We use the threshold model where asymmetry is primarily caused by regime switching. The issue whether or not disturbances follow asymmetric distributions is secondary. By contrast most findings of Psaradakis and Sola (2003) are driven by asymmetric disturbances, see their equations (6)–(8). We notice that Sichel’s definition of deepness and steepness explicitly indicates two regimes of economy (troughs vs peaks, or contractions vs expansions). Moreover, the adjustment speeds of economic variables are perceived to vary across the two

\textsuperscript{1}Psaradakis and Sola (2003) discuss briefly the logistic smooth transition autoregressive model, not TAR/MTAR models
regimes (e.g., steeper contraction implies faster adjustment speed). Therefore the threshold model, which allows for regime-dependent adjustment speeds, provides a natural formulation of the asymmetry defined by Sichel. It is straightforward to augment the threshold model with asymmetric disturbances. In this sense the threshold model may add more flexibility to the modeling of asymmetry.

The performance of the threshold test when applied to filtered data is examined by extensive Monte Carlo experiments. Special attention is paid to examining the effects of (I) the degree of regime switching, (II) the number of observations subject to regime switching and (III) the possible heterogeneity in disturbances. We consider the Hodrick-Prescott filter with various smoothness parameters, the Band Pass filter with various truncation approximations, and moving average filter of various orders. We apply these filters to the simulated data that consist of a stochastic trend component and a regime-switching cyclical component. The rejection frequencies of the threshold test are compared under different scenarios. We also provide an empirical illustration and test the cyclical asymmetry in real U.S. GNP based on the threshold model.

The Threshold Test

Let \((e_1, \ldots, e_n)\) be the cyclical components of a real economic variable. We assume \(e_t\) can be modeled by a two-regime self-exciting threshold autoregression (SETAR) of order \(p\)

\[
e_t = \left( \beta_{10} + \sum_{j=1}^{p} \beta_{1j} e_{t-j} + v_{1t} \right) 1_{1t} + \left( \beta_{20} + \sum_{j=1}^{p} \beta_{2j} e_{t-j} + v_{2t} \right) (1 - 1_{1t}),
\]

where \(v_{it}\) is the disturbance in regime \(i, i = 1, 2\). The disturbance may follow asymmetric distribution. The indicator in (1) is specified as

\[
1_{1t} = \begin{cases} 
1, & \text{if } e_{t-d} > \tau \text{ (regime 1)} \\
0, & \text{if } e_{t-d} \leq \tau \text{ (regime 2)}
\end{cases}
\]

The parameter \(\tau\) denotes the threshold value, and \(d\) denotes the delay lag. Basically model (1) describes an economy that switches across two regimes depending on whether the cyclical component is above or below the threshold value. Asymmetry is inherently accounted for by the regime-dependent autoregressive coefficients \(\beta_{ij}, (i = 1, 2, j = 0, \ldots, p)\). These coefficients also control the regime-varying adjustment speeds. Model (1) can characterize the deepness type of asymmetry defined by Sichel (1993). To capture the steepness type of asymmetry,
the indicator can be redefined as

\[ 1_{1t} = \begin{cases} 
1, & \text{if } \Delta e_{t-d} > \tau \text{ (regime 1)} \\
0, & \text{if } \Delta e_{t-d} \leq \tau \text{ (regime 2)}
\end{cases} \quad (3) \]

Now the regime switching is determined by the difference of the cyclical component. Model (1) along with (3) is the MTAR model of Enders and Granger (1998). Figure 1 shows the two types of asymmetry in simulated TAR and MTAR series.

Testing asymmetry in Model (1) amounts to testing the null hypothesis

\[ H_0 : \beta_{10} = \beta_{20}, \beta_{11} = \beta_{21}, \ldots, \beta_{1p} = \beta_{2p} \quad (4) \]

against the alternative hypothesis that at least one of the above equalities does not hold. In other words we test a linear autoregression (null model) against a threshold autoregression (alternative model). This test is special because under the null hypothesis (4) the indicator is canceled out and therefore the threshold value, \( \tau \), is not identified. In order to resolve this so-called Devies problem we use the supF test of Hansen (1996):

\[ \sup F = \sup_{\tau \in [\tau^l, \tau^u]} \frac{n [RSS1(\tau) - RSS0]}{RSS1(\tau)}, \quad (5) \]

where \( RSS1(\tau) \) is for given \( \tau \) the residual sum of squares of the threshold autoregression (1), and \( RSS0 \) is the residual sum of squares of the linear autoregression

\[ e_t = \beta_0 + \sum_{j=1}^p \beta_j e_{t-j} + v_t. \quad (6) \]

The parameter space for the threshold value is \( [\tau^l, \tau^u] \) and we assume the true threshold value lies in this interval. Following Hansen (2000) we specify \( \tau^l \) and \( \tau^u \) as the 15th and 85th percentiles of the empirical distribution of \( e_{t-d} \) (or \( \Delta e_{t-d} \) for the MTAR model).

Hansen (1996) proves that the asymptotic null distribution of the supF test is nonstandard. Hansen (1996) recommends the following bootstrap procedure to obtain the bootstrap p-value:

1. Generate a random draw \( e_t^* \sim N(0, 1), t = 1, \ldots, n \).

2. Use \( e_t^* \) as the dependent variable, and use the original independent variables in (1).

Fit the threshold autoregression and keep the residual sum of squares \( RSS1^*(\tau) \).
3. Use $e_t^*$ as the dependent variable, and use the original independent variables in (6). fit the linear autoregression and keep the residual sum of squares $RSS_0(\tau)$.

4. Compute $\sup F^* = \sup_{\tau \in [\tau', \tau'']} \frac{n[RSS_1(\tau) - RSS_0(\tau)]}{RSS_1(\tau)}$.

5. Repeat Steps 1-4 $B$ times. The bootstrap p-value is computed as the percentage of bootstrap samples for which $\sup F^*$ exceeds the observed $\sup F$.

The null hypothesis (4) is rejected if the bootstrap p-value is less than the predetermined significance level, say $\alpha = 0.05$.

Several practical issues need discussions. First, the delay lag $d$ is typically unknown. We can estimate the threshold value $\tau$ and the day lag $d$ simultaneously by two nested grid searches. See Hansen (1997) for more on this. Second, the autoregressive lag number $p$ in model (1) can be selected by information criteria such as AIC and BIC. Or we may start with a parsimonious model and increase $p$ until no serial correlation is detected in the residual. This paper selects $p$ by minimizing BIC. Third, the intercepts terms $\beta_{10}$ and $\beta_{20}$ in model (1) can be dropped if we have a priori belief. This is the case when $e_t$ is the regression residual. Finally, we can redefine the indicator to allow for more than two regimes and exogenous threshold variables. In Monte Carlo experiments we focus on the basic two-regime model (1) assuming unknown $\tau$ but known $p$ and $d$.

Filters

Data are filtered for different reasons such as signal extraction and smooth forecasting. Macroeconomists filter data in order to isolate the cyclical component out of a trending variable like GNP. We consider the following three filters$^2$.

Hodrick Prescott Filter

Assume that the observed time series $y_t$ consists of the nonstationary secular component $x_t$, and the stationary cyclical component $e_t$. The Hodrick Prescott (HP) filter obtains $e_t$ as the solution to the optimization problem:

$$
\min_{x_t} \left\{ \sum_{i=1}^{n} (y_t - x_t)^2 + \lambda \sum_{i=2}^{n-1} [(x_{i+1} - x_i)^2 - (x_i - x_{i-1})^2] \right\}
$$

(7)

$$
e_t = y_t - x_t
$$

(8)

$^2$Following Baxter and King (1999) we do not consider the Beveridge-Nelson (BN) decomposition because the link between the business cycle and BN decomposition is unclear.
where the smoothness of the secular component is controlled by $\lambda$. Hodrick and Prescott (1997) recommend $\lambda = 6.25$ for monthly data and $\lambda = 1600$ for quarterly data. King and Rebelo (1993) provide the explicit form for the cyclical component

$$e_t = H(L)y_t \equiv \left( \frac{\lambda(1 - L)^2(1 - L^{-1})^2}{1 + \lambda(1 - L)^2(1 - L^{-1})^2} \right) y_t,$$

(9)

where $H(L)$ denotes the HP filter, $Ly_t = y_{t-1}$, $L^{-1}y_t = y_{t+1}$. Formula (9) makes it clear that the nonstationary component integrated of order four or less has been removed. The frequency-response function of $H(L)$ is

$$H(e^{-i\omega}) = \left( \frac{4\lambda(1 - \cos(\omega))^2}{1 + 4\lambda(1 - \cos(\omega))^2} \right),$$

(10)

where $\omega$ denotes the frequency (radian per unit time). It is easy to show $H(0) = 0$ and $H(\pi) = 16\lambda/(1 + 16\lambda) \approx 1$. So the HP filter attenuates low frequency and provides approximation for a high pass filter. This paper does not consider the first difference filter because it provides bad approximation for the high pass filter. See figure 5-A and 5-B of Baxter and King (1999) for more discussions.

**Band Pass Filter**

The National Bureau of Economic Research (NBER) defines the business cycle as the periodic fluctuation between 6 and 32 quarters (18 and 96 months). The Band Pass (BP) filter can be used to pass through components in that interval, and remove lower and higher frequencies. The cyclical component can be obtained via the *ideal* BP filter as

$$e_t = \sum_{j=-\infty}^{\infty} b_j y_{t-j},$$

(11)

where $b_j$ is the $j$th filter coefficient. The frequency-response function of the BP filter is

$$\sum_{j=-\infty}^{\infty} b_j e^{-i\omega j} = \begin{cases} 1, & \text{if } |\omega| \in [\omega', \omega^u] \\ 0, & \text{elsewhere} \end{cases}$$

(12)
where \( \omega^l \) and \( \omega^u \) are the lower and upper bounds of the frequency band. After applying the inverse Fourier transformation to (12), the coefficient of ideal BP filter is given by

\[
b_j = \begin{cases} 
\frac{\sin(j \omega^u) - \sin(j \omega^l)}{\pi j}, & j = \pm 1, \pm 2, \ldots \\
\frac{\omega^u - \omega^l}{\pi j}, & j = 0
\end{cases}
\]

The ideal BP filter involves infinite number of coefficients. In practice an approximation for the ideal BP filter is obtained via truncation. Baxter and King (1999) propose the following truncation approximation

\[
e_t = \sum_{j=-q}^{q} \hat{b}_j y_{t-j}, \quad t = q + 1, \ldots, n - q
\]

where

\[
\hat{b}_j = b_j - (1 + 2q)^{-1} \sum_{i=-q}^{q} b_i, \quad j = 0, \pm 1, \ldots, \pm q.
\]

For quarterly data, Baxter and King (1999) recommend setting \( q = 12, \omega^l = \pi/16, \omega^u = \pi/3 \).

**Moving Average Filter**

The trend component can also be estimated as centered moving average. Then the cyclical component is obtained as the deviation of the series from the trend. Explicitly,

\[
e_t = y_t - \frac{1}{2m + 1} \sum_{j=-m}^{m} y_{t-j},
\]

where \( m \) is the order of moving average filter.

**Simulation Study**

In this section we examine the finite sample performance of the supF test (5) when it is applied to filtered data. By construction the unfiltered data \( y_t \) consist of a trend component \( x_t \) and a cyclical component \( e_t \). The trend component is modeled as a random walk with a drift term. The cyclical component is generated by the threshold autoregression. Under the null hypothesis of symmetry the threshold autoregression reduces to the linear autoregression.
The data generating process is

\[ y_t = x_t + e_t, \quad (t = 1, \ldots, 150) \] (17)

\[ x_t = d + x_{t-1} + cz_t, \quad z_t \sim \text{iidn}(0,1) \] (18)

\[ e_t = (\beta_{10} + \beta_{11}e_{t-1} + \beta_{12}e_{t-2} + v_{1t})1(e_{t-1} > \tau) + (\beta_{20} + \beta_{21}e_{t-1} + \beta_{22}e_{t-2} + v_{2t})1(e_{t-1} \leq \tau) \] (19)

We consider various parameterizations for (18)\(^3\) and (19)\(^4\). For each parameterization 500 \(y_t\) series are generated. We filter \(y_t\) using three filters. For the HP filter we consider two sets of values for the smoothness parameter, \(\lambda = \{2, 4, 6, 8, 10\}\), \(\lambda = \{1200, 1400, 1600, 1800, 2000\}\). The first set of values include the one recommended for monthly data and the second set for quarterly data. Both sets are considered here because we use the generated data rather than monthly or quarterly observations of real variables.

We use the BP filter to isolate five bands of frequencies, \(\text{band1} = [0, \pi]\), \(\text{band2} = [\pi, 2\pi]\), \(\text{band3} = [2\pi, 3\pi]\), \(\text{band4} = [3\pi, 4\pi]\), \(\text{band5} = [4\pi, \pi]\). Certain combinations of these bands may be interesting. For example the union of \(\text{band1}\) and \(\text{band2}\) contains the band recommended for the quarterly data \([\frac{1}{16}, \frac{\pi}{3}]\). To highlight the effect of truncation approximation we let \(q = 5, 10\) for the truncated BP filter (15). For the MA filter (16) we let \(m = \{1, 2, 3, 4, 5, 11, 12, 13, 14, 15\}\).

In order to examine the effect of filtering data, we apply the supF threshold test (5) to both the true cyclical component \(e_t\) and the filtered \(y_t\). The bootstrap p-value is calculated and the number of bootstrap samples is set to 200. The null hypothesis of symmetry

\[ H_0 : \beta_{10} = \beta_{20}, \beta_{11} = \beta_{21}, \beta_{12} = \beta_{22} \] (20)

is rejected if the bootstrap p-value is less than the significance level 0.05. First we consider the parameter set

\[(\text{DGP1})\ d = 0.01, c = 0.01, \beta_{10} = \beta_{20} = 0.2, \beta_{11} = \beta_{21} = 0.2, \beta_{12} = \beta_{22} = 0.1, v_{1t} \sim \text{iidn}(0,1), v_{2t} \sim \text{iidn}(0,1).\]

Under (DGP1) the cyclical component is generated by a linear autoregression. Thus the null hypothesis of symmetry is imposed. Figure 2 plots the rejection frequency (size) of

\(^3\)We refer to Watson (1986) for the specification of the stochastic trend component.

\(^4\)A TAR model is specified by (19). In preliminary studies we also consider the MTAR model, and no qualitatively different results are found.
the supF test. The left panel of Figure 2 is concerned with the HP filtered data. The rejection frequency is on the vertical axis and the smoothness parameter is on the horizontal axis. The line with triangles denotes the rejection frequency of the supF test when data are filtered by the HP filter with $\lambda = 2, 4, 6, 8, 10$. The line with boxes is when $\lambda = 1200, 1400, 1600, 1800, 2000$. The line with circles is when the supF test is applied to the true cyclical component $e_t$. This line is horizontal by construction (because it is independent of $\lambda$). The middle panel of Figure 2 displays the rejection frequencies in band1, band2, band3, band4, band5 where data are filtered by the BP filter with $q = 5$ (the line with triangles) and $q = 10$ (the line with boxes). The right panel presents the rejection frequencies when data are filtered by the MA filter with $m = 1, 2, 3, 4, 5$ (triangles) and $m = 11, 12, 13, 14, 15$ (boxes).

The reported rejection frequencies in Figure 2 are all close to the significance level of 0.05 except for the BP filter with $q = 10$ in band4. There are small evidences that the supF test is oversized when applied to the true cyclical component but undersized when applied to the filtered data. Overall in terms of sizes, filtering data has limited effects on the performance of the supF test. Next we examine the power of the supF test and consider

\[(DGP2)\ d = 0.01, c = 0.01, \tau = 0, \beta_{10} = 0.2, \beta_{20} = -0.8, \beta_{11} = 0.2, \beta_{21} = 0.5, \beta_{12} = 0.1, \beta_{22} = -0.1, v_{1t} \sim iidn(0, 1), v_{2t} \sim iidn(0, 1).\]

Now the cyclical component is generated by a threshold autoregression with regime-varying intercepts and autoregressive coefficients. The rejection frequency (power) of the supF test is displayed in Figure 3A. The power is slightly above 0.7 when the supF test is applied to the true cyclical component. This finding is consistent with Hansen (1996). When the data are filtered by the HP filter, the power is an increasing function of $\lambda$ when $\lambda$ is small, and becomes stable at level of around 0.5 when $\lambda = 1X00$. For the BP filter, the power is extremely low except in band4 and $q = 10$. But even in this case the power is less than 0.4. The power for the MA filter is slightly less than that for the HP filter.

To see the effect of filtering data from a different perspective, Figure 3B shows the un-smoothed periodogram\(^5\) for the true cyclical component (the leftmost panel), and the data filtered by the HP filter with $\lambda = 1600$, the BP filter with $q = 10$ in band4 and the MA filter with $m = 13$ (the rightmost panel). It is clear that the periodogram for the HP filtered data

\(^5\)Let $\omega = 2\pi j/n, (j = 0, \ldots, n - 1)$ be the Fourier frequency. The Fourier transformation of $x_t$ is given by $f(e^{-i\omega}) = \sum_{t=1}^{n} x_t e^{-i\omega t}$. The un-smoothed periodogram is calculated as $f(e^{-i\omega})f(e^{i\omega})/n$.
has the highest resemblance to the unfiltered data. The location and height of the spikes in the two panels correspond relatively well. The discrepancy exists only for the frequencies near 0. The periodogram for the MA filtered data has less resemblance (pay attention to the relative height of spikes). Finally the periodogram for the BP filtered data looks remarkably different from the unfiltered data. Only two shrunk spikes in the unfiltered data are captured by the BP filter; other frequencies are attenuated. This fact is caused by the band-passing nature and the truncation approximation.

The finding in Figure 3B is consistent with Figure 3A. Basically filtering data introduces distortion. This can be seen from the power deduction in Figure 3A, and the different patterns in the periodogram in Figure 3B. In terms of power, the HP filter has the smallest effect on the performance of the supF test. This comes as no surprise given the close resemblances between its periodogram and the unfiltered data. Next we investigate what if the regime-switching becomes less evident. We consider

\[ (DGP3) \quad d = 0.01, c = 0.01, \tau = 0, \beta_{10} = 0.2, \beta_{20} = -0.2, \beta_{11} = 0.2, \beta_{21} = 0.3, \beta_{12} = 0.1, \beta_{22} = -0.1, v_{1t} \sim \text{iidn}(0, 1), v_{2t} \sim \text{iidn}(0, 1). \]

For easy exposition we underline the difference between the new DGP and (DGP2). In this case the differences between \( \beta_{10} \) and \( \beta_{20} \), \( \beta_{11} \) and \( \beta_{21} \) in (DGP3) are smaller than (DGP2). From Figure 4 we see that the power of the supF test decreases significantly. This is because the alternative model moves closer to the null model. The ranking of the power lines remains unchanged in Figure 4 though. Power deduction can also be seen in Figure 5, which is concerned with

\[ (DGP4) \quad d = 0.01, c = 0.01, \tau = 0.5, \beta_{10} = 0.2, \beta_{20} = -0.8, \beta_{11} = 0.2, \beta_{21} = 0.5, \beta_{12} = 0.1, \beta_{22} = -0.1, v_{1t} \sim \text{iidn}(0, 1), v_{2t} \sim \text{iidn}(0, 1). \]

The greater threshold value \( \tau \) in (DGP4) reduces the number of observations subject to regime switching (or the likelihood of regime switching). Consequently the power of the supF test falls. We can imagine when \( \tau \rightarrow \infty \) the supF test will lose all power since the threshold autoregression will be dominated by the linear autoregression in the regime specified by \( 1(e_{t-1} < \tau) \). Comparison of Figure 3A and Figure 5 illustrates that the increasing \( \tau \) indeed lowers the power of the supF test. Next we investigate how the trend component affects the power.
\[ (DGP5) \quad d = 0.02, c = 0.5, \tau = 0, \beta_{10} = 0.2, \beta_{20} = -0.8, \beta_{11} = 0.2, \beta_{21} = 0.5, \beta_{12} = 0.1, \beta_{22} = -0.1, v_{1t} \sim iidn(0, 1), v_{2t} \sim iidn(0, 1). \]

Compared to (DGP2) the new drift term \( d = 0.02 \) increases the growth rate of the trend component. The new standard deviation \( c = 0.5 \) adds more volatility to the trend component. We conjecture that the increased variation in the trend component would worsen the performance of all filters, since it becomes harder to distinguish the cycle from the trend volatility. This conjecture is confirmed by Figure 6. Comparing Figure 6 to Figure 3A, we see the power for unfiltered data remains unchanged (because no changes are made to \( e_t \)). However, the power for the filtered data deteriorates substantially. In particular, the maximum power for the HP filter and the MA filter is now less than 0.2. There is almost \((0.5 - 0.2)/0.5 = 60\%\) deduction in power.

So far we may form an impression that filtering data can only lowers the power. This is also the main finding in Psaradakis and Sola (2003). Figure 7 provides an interesting counter-example showing that in some cases filtering data can enhance the power. The DGP used by Figure 7 is

\[ (DGP6) \quad d = 0.01, c = 0.01, \tau = 0, \beta_{10} = 0.2, \beta_{20} = -0.8, \beta_{11} = 0.2, \beta_{21} = 0.5, \beta_{12} = 0.1, \beta_{22} = -0.1, v_{1t} \sim iidn(0, 1.5^2), v_{2t} \sim iidn(0, 0.5^2). \]

Basically (DGP6) allows for regime-varying disturbances \( v_{1t} \) and \( v_{2t} \). In this case the disturbance variance switch across regimes. The heteroskedasticity has two impacts on the power of the threshold test. First, heteroskedasticity adds more variations across regimes, and therefore improves the power (for both unfiltered and filtered data). Second, heteroskedasticity may increase the volatility in the cyclical component relative to the trend component. Then all filters perform better because of the increased signal to noise ratio (or cycle to trend ratio). We can easily spot the power improvement for the filtered data by comparing Figure 7 to Figure 3A. In particular the maximum powers for the HP filtered data and MA filtered data in Figure 7 are greater than the power for the unfiltered data.

**U.S. GNP**

In this section we test the asymmetry in the cyclical component of log U.S. real GNP using the threshold model (1). The quarterly data from 1949Q1 through 1984Q1 are obtained
from Economic Data - FRED. We filter the data using the HP filter (9) with $\lambda = 1600$, and the truncated BP filter (15) with $q = 12$. The frequency bound is set as $\omega^l = \pi/16, \omega^u = \pi/3$ for (13). We also use the MA filter (16) with $m = 13$ given the finding in Figure 3A. The unfiltered and filtered data are plotted in Figure 8A, in which the filtered data appear stationary. Figure 8A also exhibits visual signals for asymmetry. For example, the red box in Figure 8A highlights a portion of the BP filtered GNP, and that portion clearly shows steeper contraction than expansion.

To examine the cyclical asymmetry, we fit the TAR model (1) along with indicator (2) for the filtered GNP. The results are summarized in panel A of Table 1. We endogenously select the threshold value $\tau$, the delay lag $d$ and the autoregressive lag $p$ by minimizing BIC$^6$. The maximum autoregressive lag $p^{\text{max}}$ is set to 8 (two years). We then apply the supF test (5) based on the selected model and simulate its bootstrap $p$-value. To ensure the model adequacy we conduct the Ljung-Box Q tests with 4 and 8 lags for the residual$^7$. Finally we report the standard deviation for the residual in regime 1 and 2. The whole process is repeated for the MTAR model that uses indicator (3), and the results are reported in panel B of Table 1. In table $1$ GNP$^i$ denotes the GNP filtered by the filter $i$, ($i = \text{HP, BP, MA}$).

We find limited evidences for cyclical asymmetry. The null hypothesis of symmetry is rejected at level 0.05 by the BP filtered GNP in the TAR model and the HP filtered GNP in the MTAR model. The bootstrap $p$-values are 0.039 and 0.017 respectively. In order to guard against spurious testing results, Figure 8B and 8C plot the residual sum of square (RSS) against the possible threshold value for the TAR model and MTAR model. For a two-regime threshold process the RSS plot would look like letter V, because the RSS is minimized by the true threshold value, cf., Hansen (2000). For a linear AR process the RSS series would fluctuate randomly. In Figure 8B and 8C only the BP filtered GNP yields the V-shaped RSS plots. The troughs in other RSS plots are more likely to be produced by randomness.

However, the existing evidence provided by the BP filtered GNP should be taken seriously. The Monte Carlo experiment in last section shows that filtering data typically leads to low power of the threshold test. Furthermore, given that $\sigma_1$ and $\sigma_2$ in Table 1 are close, the power for the filtered data is unlikely to exceed that for the unfiltered data. Therefore the given evidence may strongly indicate true asymmetry in the cyclical component of GNP. Psaradakis and Sola (2003) give similar guidance about interpreting the test results for

---

$^6$BIC = log($\text{RSS}/n$) + $k \log(n)/n$, where RSS denotes the residual sum of squares, $n$ is the sample size and $k$ is the number of parameters. The AIC criterion gives similar results.

$^7$We also consider the Breusch–Godfrey LM test for no autocorrelation in residuals and obtain similar results.
filtered data.

**Conclusions**

This paper investigates the effects of filtering data on the performance of the threshold test. Monte Carlo experiments show that filtering data lowers the power of the threshold test in most cases. This finding is consistent with Psaradakis and Sola (2003). Second, the size of the threshold test is largely unaffected by the filters. Third, the Hodrick-Prescott filter and the Moving Average filter have similar effects on the power of the threshold test. Fourth, the power of the threshold test is sensitive to the filter specification such as the smoothness parameter for the Hodrick-Prescott filter. Fifth, the specification of the trend component and cyclical component have important impacts on the power. We provide an interesting example showing that heterogeneity in the disturbances may lead to greater power for the filtered data than the unfiltered data. Finally we apply the threshold test to the U.S. real GNP. Evidences for cyclical asymmetry emerge when GNP is filtered by the Band Pass filter in the TAR model and the Hodrick-Prescott filter in the MTAR model.
References


Davies, R. B. (1977). Hypothesis testing when a nuisance parameter is present only under the alternative. *Biometrika*, 64, 247–254.

Davies, R. B. (1987). Hypothesis testing when a nuisance parameter is only identified under the alternative. *Biometrika*, 74, 33–43.


Figure 1: Simulated TAR and MTAR series.

The DGP for the TAR model is 
\[
e_t = (-0.1 + 0.9e_{t-1})1(e_{t-1} < 0) + (0.2 + 0.1e_{t-1})1(e_{t-1} \geq 0) + 0.5v_t, \\
v_t \sim \text{iidn}(0,1), t = (1, \ldots, 60).
\]

The DGP for the MTAR model is 
\[
e_t = (-0.1 + 0.9e_{t-1})1(\Delta e_{t-1} < 0) + (0.2 + 0.1e_{t-1})1(\Delta e_{t-1} \geq 0) + 0.5v_t, \\
v_t \sim \text{iidn}(0,1), t = (1, \ldots, 60).
\]
Figure 2: Rejection frequencies of the supF test for DGP1
Figure 3A: Rejection frequencies of the supF test for DGP2
Figure 3B: Periodogram for the unfiltered and filtered data
Figure 4: Rejection frequencies of the supF test for DGP3
Figure 5: Rejection frequencies of the supF test for DGP4
Figure 6: Rejection frequencies of the supF test for DGP5
Figure 7: Rejection frequencies of the supF test for DGP6
Figure 8A: Plots of Log Real U.S. GNP and Filtered GNP
Figure 8B: RSS Plot, the TAR model
Figure 8C: RSS Plot, the MTAR model
Table 1: Testing Cyclical Asymmetry in Log Real U.S. GNP

### Panel A: TAR Model

<table>
<thead>
<tr>
<th></th>
<th>p</th>
<th>d</th>
<th>( \tau )</th>
<th>sup F</th>
<th>p-value</th>
<th>( Q(4) )</th>
<th>( Q(8) )</th>
<th>( \sigma_1 )</th>
<th>( \sigma_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>GNP(^{HP})</td>
<td>2</td>
<td>1</td>
<td>-0.016</td>
<td>11.66</td>
<td>0.141</td>
<td>2.845</td>
<td>7.368</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>GNP(^{BP})</td>
<td>6</td>
<td>3</td>
<td>0.007</td>
<td>26.50*</td>
<td>0.039</td>
<td>0.506</td>
<td>4.761</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>GNP(^{MA})</td>
<td>3</td>
<td>1</td>
<td>-0.016</td>
<td>9.762</td>
<td>0.463</td>
<td>0.652</td>
<td>1.129</td>
<td>0.071</td>
<td>0.010</td>
</tr>
</tbody>
</table>

### Panel B: MTAR Model

<table>
<thead>
<tr>
<th></th>
<th>p</th>
<th>d</th>
<th>( \tau )</th>
<th>sup F</th>
<th>p-value</th>
<th>( Q(4) )</th>
<th>( Q(8) )</th>
<th>( \sigma_1 )</th>
<th>( \sigma_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>GNP(^{HP})</td>
<td>2</td>
<td>2</td>
<td>-0.011</td>
<td>16.89*</td>
<td>0.017</td>
<td>1.478</td>
<td>5.087</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>GNP(^{BP})</td>
<td>6</td>
<td>2</td>
<td>0.005</td>
<td>24.56</td>
<td>0.086</td>
<td>0.677</td>
<td>4.379</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>GNP(^{MA})</td>
<td>2</td>
<td>2</td>
<td>-0.010</td>
<td>11.10</td>
<td>0.171</td>
<td>0.149</td>
<td>1.514</td>
<td>0.008</td>
<td>0.010</td>
</tr>
</tbody>
</table>

This table reports the test for asymmetry in the log real quarterly U.S. GNP from 1949Q1-1984Q1 filtered by the HP filter with \( \lambda = 1600 \) (\( GNP^{HP} \)), the approximated BP filter filter with \( q = 12, \omega^j = \pi/16, \omega^u = \pi/3 \) (\( GNP^{BP} \)) and the MA filter with \( m = 13 \) (\( GNP^{MA} \)) in the TAR model (panel A) and MTAR model (panel B). We report the selected lag \( p \); delay lag \( d \); the threshold value \( \tau \); the supF test, its bootstrap \( p \)-value; the Ljung-Box Q test with 4 lags \( Q(4) \) and 8 lags \( Q(8) \) for the residuals; and the standard deviation of the residual in regime 1 (\( \sigma_1 \)) and 2 (\( \sigma_2 \)). * denotes significance at level of \( \alpha = 0.05 \).