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Interest Rates, Bond Sales, and the IS-LM Model

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I. Introduction

In recent years there has been heightened concern among professional economists and in the financial community regarding the potential macroeconomic consequences of substantial and persistent government deficits.\(^1\) This concern has centered on the interest rate effect of increased supplies of government bonds required to finance the deficit and the consequent crowding out of private investment expenditures that slows economic growth and lowers future living standards. At the same time, and in sharp contrast to these concerns, is Barro’s [1974, 1979, 1989] revival of the Ricardian Equivalence Theorem which, in its strongest form, states that government spending financed by bond sales has the same real effects as government spending financed by taxes. This implied equivalence of bond sales and taxes suggests that it is the level of government spending itself that determines the impact on real variables (e.g., real GDP) and not the means by which that spending is financed.

Given the significance of these issues and the surrounding controversy, one would expect the leading textbooks in macroeconomic theory to contain a careful analysis of the effect of increased bond supplies on the interest rate and a discussion of the conditions under which these bond sales will, or will not, crowd out private spending. Surprisingly, quite the opposite is true. When analyzing the short-run effects of bond-financed fiscal operations over the cycle, most texts ignore this issue altogether, focusing instead on the interest rate effect resulting from the change in real output following the fiscal operation.

\(^1\) In 2000, the federal government ran a budget surplus of $236.2 billion. Following the 2001 recession and the Bush tax cuts, the government budget shifted to a $157.8 billion deficit in 2002 followed by deficits of $377.6 billion in 2003, $412.7 billion in 2004, $318.3 billion in 2005, and $248.2 billion in 2006. [Source: Historical Budget Data, Congressional Budget Office].
Our purpose in this paper is to sort through these issues, pointing out those areas where the discussion of the economic consequences of government bond sales to finance a rise in government spending (or a tax cut) is confusing, incomplete, and, in some cases, incorrect. Throughout the analysis, we employ the IS-LM framework as it provides the most convenient pedagogical and conceptual vehicle for a unified discussion of these issues.

II. The Basic Model

The basic model that appears in most textbooks is the fixed-price, end-of-period IS-LM framework with wealth, however defined, absent from both the consumption and money demand functions. Algebraically, this model is:

\[
\text{(IS)} \quad y = c(y - t) + i(r) + g \\
\text{(LM)} \quad m = m^d(y, r),
\]

where: \(y\) is output, \(t\) is tax revenues, \(r\) is the (real) interest rate, \(c(y-t)\) is desired consumption, \(i(r)\) is desired investment, \(g\) is government purchases of goods and services, \(m^d(y, r)\) is the real demand for money, \(m = M/P\) is the real money supply, and \(P\) is the (fixed) price level. While generally not explicit, certainly implicit in this model is the government budget constraint requiring government purchases of goods and services to be financed by some combination of taxes, bond sales, and money creation.\(^2\)\(^3\)

\[
g \equiv t + \frac{\Delta B^g}{rP} + \frac{\Delta M}{P}.
\]

\(^2\) We assume that each government bond is a consol paying $1/year in interest so that the market price can be written compactly as $1/r. \(B^g\) is the number of government bonds outstanding. To simplify, we ignore interest payments on the outstanding stock of government bonds (i.e., on the accumulated debt).

\(^3\) There are two additional budget constraints in the model, the economic consequences of which are frequently overlooked when analyzing the macroeconomic effects of bond-financed fiscal operations.
In the basic model, a rise in \( g \) unaccompanied by a change in either \( M \) or \( t \) necessarily implies debt financing. This result is illustrated in Figure 1. Starting from equilibrium at point A, where \( y_1 \) is natural output, a bond-financed rise in government expenditures from \( g_1 \) to \( g_2 \) shifts the IS curve to the right. The new short-run equilibrium is given by point C.\(^4\)

The standard explanation of the adjustment dynamics is as follows. The rise in \( g \) increases aggregate demand and, via the simple Keynesian multiplier, causes output to rise. Mathematically, the rise in \( g \) causes output to rise by:

\[
\frac{\partial y}{\partial g} = \left[ \frac{1}{(1-c) + m^d (i^d / m^d)} \right] [dg],
\]

where \( c = \frac{\partial c}{\partial y} \) is the marginal propensity to consume, \( i_r = \frac{\partial i}{\partial r} \) the interest responsiveness of investment, \( m^d = \frac{\partial m^d}{\partial y} \) is the income responsiveness of money demand, and \( m^d_r = \frac{\partial m^d}{\partial r} \) is the interest responsiveness of money demand.

---

\(^4\) Mathematically, the rise in \( g \) causes output to rise by:

\[
\frac{\partial y}{\partial g} = \left[ \frac{1}{(1-c) + m^d (i^d / m^d)} \right] [dg],
\]

where \( c = \frac{\partial c}{\partial y} \) is the marginal propensity to consume, \( i_r = \frac{\partial i}{\partial r} \) the interest responsiveness of investment, \( m^d = \frac{\partial m^d}{\partial y} \) is the income responsiveness of money demand, and \( m^d_r = \frac{\partial m^d}{\partial r} \) is the interest responsiveness of money demand.
rise by: \( y_3 - y_1 = \frac{1}{1 - c_y} \cdot [\Delta g] \). In Figure 1, this corresponds to the movement from point A to point B. At point B, the goods market is once again in equilibrium, as indicated by the fact that the economy is back on the IS curve. The money market, however, is now out of equilibrium. The rise in output to \( y_3 \) increases the transactions demand for money \( (since m^d_y > 0) \), inducing the private sector to sell bonds in a vain attempt to build its money holdings. The price of bonds falls, and \( r \) rises, causing investment demand, hence output, to fall back somewhat from \( y_3 \) to \( y_2 \). The final equilibrium is at point C, where the demand for money has necessarily returned to its original level.\(^5\) The important point for our purposes is that the rise in \( r \) is fully explained by the increase in \( y \) that logically precedes it.

What some may find puzzling about this analysis is that the increased stock of government bonds sold to finance the rise in \( g \) has no independent effect on the rate of interest.\(^6\) No increase in \( r \) over and above the change resulting from the rise in output is required to induce the private sector to absorb the larger stock of government bonds now

\(^5\) In many texts this dynamic process is described as a “stairstep” process that causes the economy to “slide” up the LM curve. As soon as \( y \) starts to rise, the resulting excess demand for money creates an excess supply of bonds and a rising interest rate. Thus the interest rate and real output rise simultaneously. While the comparative static results are identical, output does not overshoot its new equilibrium value of \( y_2 \) in this alternate dynamic interpretation.

\(^6\) A simple numerical example may serve to better illustrate this result. Assume the rise in \( g \) is $10, \( c_y = 0.75 \) so that the simple Keynesian multiplier is 4, and \( m^d_y = 0.2 \). If \( g \) rises by $10, the initial rise in \( y \) prior to the rise in \( r \) (the rise in \( y \) to \( y_3 \) in Figure 1) will be $40 which, in turn, increases the demand for money by $8. This $8 excess money demand (excess bond supply) is the source of the upward pressure on \( r \) in the basic IS-LM model. In the basic model, then, this $8 (income-induced) decrease in the demand for bonds by the private sector exerts upward pressure on the interest rate, while, in the same analysis, a $10 increase in the supply of bonds required to finance the rise in \( g \) has no effect whatsoever.
outstanding. This is certainly contrary to what we know, or think we know, about the effect of an increase in supply on market price (the price of bonds in this instance).

The basic IS-LM model described by equations 1 and 2 is the approach employed by most of the leading macroeconomic textbooks to analyze the consequences of a bond-financed increase in $g$. Abel, Bernanke, and Croushore [2008], Blanchard [2006], Dornbusch, Fischer, & Startz [2008], Froyen [2005], Gordon [2006], and Mankiw [2007] all attribute the rise in $r$ to the increased transactions demand for money that accompanies the rise in output. And yet, while their explanations are technically correct, each fails to mention, much less explain, an assumption embedded in equation 2 (which we discuss below) that prevents the increased supply of government bonds from exerting an independent effect on the interest rate.

The context of the one-period result presented above ($A \rightarrow B \rightarrow C$ in Figure 1) is that of a fixed-price model in which the level of output is entirely determined by aggregate demand (corresponding to the intersection of the IS and LM curves). In the long run, however, the excess demand generated by the bond-financed rise in $g$ raises the price level, shifting the LM curve left until a new long-run equilibrium is reached at point D in Figure 1. The further increase in $r$ from $r_2$ to $r_3$ is fully accounted for by the effect of the rising price level on the money market. The rising price level raises the (nominal) demand for money and, once again, causes private sector agents to attempt to reallocate their financial portfolio away from bonds and towards money. The price level, and

---

7 In effect, the increased supply of government bonds to finance the higher $g$ is absorbed into private-sector portfolios at the existing interest rate. The reason for this unusual result is, as we show below, that the basic IS-LM model incorporates an extreme assumption about the relationship between wealth and the demand for bonds. Specifically, an increase in wealth raises the demand for bonds by the rise in wealth. Consequently, when the government sells bonds to finance an increase in $g$, the resulting rise in wealth raises the demand for bonds by the same amount as the increase in the supply of bonds. Hence, no independent change in the interest rate is required.
therefore $r$, must continue to rise until interest-elastic private spending (investment demand in the basic model) has been reduced by the same amount as the rise in $g$. In other words, crowding out is complete in the long run. But still there is no independent effect of government bond sales on $r$ notwithstanding the fact that the stock supply of these bonds is increasing period-by-period (assuming that the increase in $g$ is permanent). The additional rise in $r$ (from $r_2$ to $r_3$) is fully accounted for by the rise in $P$.

This brief overview of the effects of a debt-financed rise in $g$ raises several interesting questions. One, what is missing in the basic IS-LM model that prevents it from capturing an independent effect of government bond supplies on the interest rate? Two, how can the model be altered to capture an independent effect? And three, what are the macroeconomic consequences for output, the interest rate, and the crowding out of private expenditures in a model in which increased supplies of government bonds have an independent effect on the interest rate? It is to these questions that we now turn our attention. For simplicity, we continue to employ the sticky-price IS-LM model in which the price level is fixed, in the short run, at $P$.

III. Wealth Effects in the Basic Model

In any period $t$, economic agents have a stock of real financial wealth, $a_t$, equal to the stock of real financial wealth inherited from the previous period plus the flow of saving (the change in real wealth) over the period, or:

$$a_t = a_{t-1} + s_t$$

in which $a_{t-1}$ is real financial wealth at the end of the previous period, and $s_t$ is the desired flow of saving in period $t$. This wealth must be held in some form in private sector asset portfolios. And since the only financial assets are money and bonds
(government plus private), the model necessarily implies a financial wealth constraint (FWC) requiring that the demand for money plus the demand for bonds equals total real financial wealth. Moreover, since total real financial wealth consists of the real supply of money plus the real supply of bonds, the FWC requires:

\[
\frac{M_t}{P} + \frac{B_t}{rP} = a_t + m_t^d(t) + b_t^d(t)
\]  

(5)

in which \( m_t^d(t) \) is the real demand for money in period \( t \), and \( b_t^d(t) \) is the real demand for bonds in period \( t \).^8

The FWC is one of two consistency relationships (the other being the budget constraint) required of all well-specified macroeconomic models. Just as the budget constraint imposes a set of adding-up conditions on the parameters of the flow variables of the model,^9 the FWC imposes a set of adding-up conditions on the parameters of the money and bond demand equations (i.e., on the stock variables).^10 Specifically, these adding-up conditions are:

\[
m_a^d + b_a^d \equiv 1
\]  

(6)

\[
m_r^d + b_r^d \equiv 0
\]  

(7)

\[
m_y^d + b_y^d \equiv 0
\]  

(8)

^8 The (real) supply of money in period \( t \) is equal to the supply inherited from the previous period, \( M_{t-1}/P \), plus any change in the money supply, \( \Delta M_t/P \), engineered by the Fed in period \( t \). Similarly, the stock supply of government and private bonds in period \( t \) equals the stock on hand at the end of the previous period, \( b_{t-1} \), plus the flow supply of government and private bonds in period \( t \). The flow supply of government bonds, \( \Delta Bg/rP \), is determined by the government budget constraint, while the flow supply of private bonds, \( \Delta Bp/rP \), is determined by the firm budget constraint. For details, refer to footnote 3.

^9 For example, the requirement that the marginal propensity to consume plus the marginal propensity to save equals 1.0 is a restriction (or adding-up condition) imposed by the household budget constraint on the consumption-saving decision of households.

^10 To our knowledge Brainard and Tobin [1968] were the first to explicitly recognize the adding-up conditions implied by the financial wealth constraint.
in which the as yet undefined parameters are: \( m^d_a = \frac{\partial m^d}{\partial a_t} \), \( b^d_a = \frac{\partial b^d}{\partial a_t} \), 

\[ b^d_r = \frac{\partial b^d}{\partial r} \text{, and } b^d_y = \frac{\partial b^d}{\partial y}. \]

The restrictions implied by equations 6, 7, and 8 enforce consistent behavior on the part of wealth holders in the private sector. Equation 6 requires the private sector to absorb all increases in wealth into their portfolios. If financial wealth rises by $1 (due to desired saving of $1 over the period), the increased demand for money (\( m^d_a \)) plus the increased demand for bonds (\( b^d_y \)) must sum to $1. By contrast, equations 7 and 8 govern the composition of the private sector’s financial wealth portfolio. More specifically, holding wealth constant, any change that, say, raises the demand for money must come at the expense of an equal reduction in the demand for bonds. A ceteris paribus decrease in the interest rate, for example, that increases the demand for money (since \( m^d_r < 0 \)) must come at the expense of an equal reduction in the demand for bonds (since \( b^d_r = -m^d_r \)).\(^{11}\)

The same is true for a ceteris paribus change in income. An increase in income, holding wealth constant, raises the demand for money (since \( m^d_y > 0 \)) at the expense of the demand for bonds (since \( b^d_y = -m^d_y \)).\(^{12}\)

It should now be apparent why the basic IS-LM model fails to capture an independent bond supply effect. The money demand function in the basic model [i.e.,

\[^{11}\text{Here we are ignoring, for simplicity, any interest-induced change in the size of the financial wealth portfolio.}\]

\[^{12}\text{This adding-up condition may be the most difficult to grasp. An increase in income affects both the desired composition and the desired size of the private sector’s financial wealth portfolio. The composition effect is the effect of the change in income on the desired demand for money and bonds holding wealth constant. This is the effect captured by equation 8. The size effect is the impact of the change in income over time on financial wealth due to the income-induced change in desired saving. Equation 6 ensures that this increase in wealth will be absorbed into private sector asset portfolios.}\]
does not include total financial wealth \((a_t)\) as an argument. It follows directly that \(m_d^d = 0\) which, via equation 6, implies \(b_d^d = 1\). Therefore, an increase in the supply of government bonds required to finance a higher level of \(g\) (or, more generally, a deficit) automatically generates an equal increase in the demand for bonds. And since the bond supply and bond demand curves shift right by the same amount there cannot be any independent “bond supply effect” on the interest rate in the basic IS-LM model.

This outcome is illustrated in Figure 2 in which the economy is initially in equilibrium at point A in panel (a). At this initial equilibrium, aggregate output \((y_1)\) equals aggregate demand \((c + i + g)\), and saving \((s)\) equals investment \([i(r_1)]\) plus the government deficit \((g_1-t_1)\). In the money market, panel (b), equilibrium is at point A where the real demand for money equals the real money supply. In the bond market, panel (c), the real supply of bonds equals the real demand for bonds at point A. The real supply of bonds is the outstanding stock of private and government bonds at the end of the previous period \((b_0 = \frac{B_o^s}{rP})\) plus the real flow supply of bonds during the period \((\Delta b^s)\) required to finance desired investment expenditures \([i(r_1)]\) and the government deficit \((g_1-t_1)\).

To isolate the effect of an increased supply of government bonds in the basic IS-LM model, we hold government spending and taxes constant at \(g_1\) and \(t_1\), respectively. In addition, we assume that \(g_1 > t_1\) and that the excess government spending is financed by bond sales. We then ask what happens in the following period (i.e., in \(t+1\)). Since \(g\) and \(t\) are unchanged in period \(t+1\), the IS curve remains stationary at its initial position in
panel (a). The LM curve must remain stationary as well, for reasons that can be seen in panels (b) and (c). In \( t+1 \), the supply of bonds must increase to finance that period’s desired investment expenditures plus the government deficit. Since investment demand and the government deficit are the same in period \( t+1 \) as in period \( t \), the bond supply curve will shift to the right by \( \Delta b_{t+1}^d = b^d_{t+1}(y, r, a_t); \quad a_t = a_o + s \)

\[ b_{t+1}^d = b^d_t(y, r, a_{t+1}); \quad a_{t+1} = a_t + s \]

\[ \Delta b_{t+1}^d = \Delta b_{t+1}^s \]

\[ b_{t+1}^s = b_t^s + i(r_t) + (g_t - t_t) \]

\[ b_t^s = b_o + i(r_t) + (g_t - t_t) \]

As regards the demand for bonds, the flow of saving increments wealth, and, since \( s = i(r) + (g - t) \), it follows that the period \( t+1 \) change in financial wealth (\( \Delta a_{t+1} \))

13 Indeed, these are two ways of saying the same thing. If \( y = c + i + g \) and \( c = y - t - s \) then \( y = (y - t - s) + i + g \), from which it follows directly that \( s = i + (g - t) \).
necessarily equals $\Delta b^s_{t+1}$, the change in the total supply of bonds. This increase in wealth must be absorbed into private sector portfolios, and the amount absorbed by increased bond demand is determined by $b^d_a$ which, in the basic IS-LM model, equals 1. Accordingly, bond demand shifts right in panel (c) by $\Delta a_{t+1} = s = \Delta b^s_{t+1}$ to point B. Since both curves shift right by the same amount, there is no effect on the interest rate.

Increased supplies of government bonds to finance a deficit (or, for that matter, private bonds sold to finance investment) are absorbed into private sector portfolios without generating any upward pressure on the interest rate. Finally, there is no impact in the money market, hence no effect on the LM curve. The money supply is exogenous and unchanged in period $t+1$, so that the money supply curve remains stationary. The money demand curve remains stationary in period $t+1$ because money demand is not affected by wealth accumulation (i.e., because $m^d_a = 0$).

What if there is a rise in $g$ in period $t+1$ requiring that even more bonds be sold to finance the (now higher) deficit? In terms of an impact on $r$ arising from the increasing supply of bonds to finance the higher $g$, the result will be the same as in panel (c) of Figure 2. That is, there will be no effect. The initial impact of the rise in $g$ is to increase output by $\Delta y = [1/(1-c_y)]g$, which increases saving by $(1-c_y)\Delta y = Ag$.\(^\text{14}\) Since the rise in $g$ is bond financed, the supply of government bonds must increase by the same as $g$ (i.e., $\Delta b^s = Ag$). On the demand side of the bond market, the rise in saving increases both financial wealth and the demand for bonds by $\Delta g$ (since $b^d_a = 1$). Once again, the bond demand and supply curves shift right by the same amount, leaving the interest rate

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\(^{14}\) As discussed earlier, this increase in $y$ is equal to the horizontal shift in the IS curve.
unchanged, ceteris paribus. While it is true that the interest rate will be higher in period $t+1$ if $g$ is higher in $t+1$, the cause of this increase in the basic IS-LM model is not the increased supply of government bonds but rather the desire by households to substitute money for bonds as output rises. In Figure 2, this output-induced effect would correspond to a rightward shift in the money demand curve in panel (b) that is matched by an equal leftward shift in the bond demand curve in panel (c).

IV. Capturing the Direct Impact of an Increased Supply of Bonds

The simplest modification of the basic model that permits an independent bond supply effect is an expansion of the money demand function to include a second “scale variable,” namely total real financial wealth ($a_t$). With this change, the money demand function becomes:

$$m^d = m^d(y, r, a)$$

in which the partial derivative of real money demand with respect to total real financial wealth ($\frac{\partial m^d}{\partial a}$) is assumed to lie between zero and one. With this specification, any increase in total real financial wealth raises both the real demand for money and the real demand for bonds, but neither increases by as much as $a_t$. Therefore, an increase in the supply of government bonds to finance a rise in $g$ (or, for that matter, to finance an on-going deficit) generates an excess supply (demand) in the bond (money) market which then exerts independent upward pressure on the interest rate. We refer to the inclusion of total financial wealth in the money demand function as the portfolio effect.

The expanded IS-LM model now consists of three equations:

$$(IS) \quad y = c(y - t) + i(r) + g$$
Equation 10 is the basic IS curve, equation 11 is the expanded LM curve, while equation 12 defines total real financial wealth (TFW).\textsuperscript{15}

The first important difference implied by this re-specification of the IS-LM model is the slope of the LM curve itself. In the basic IS-LM model, the slope of the LM curve is:

\[
\frac{dr}{dy}_{LM} = -\frac{m^*_y}{m^*_r} > 0
\]  

(13)

By contrast, the slope of the expanded LM curve, equation 11, is:

\[
\frac{dr}{dy}_{LM} = -\frac{\left(m^*_y + m^*_y [1 - c_x]\right)}{m^*_r} > 0
\]  

(14)

Comparing equation 13 with equation 14, we see that the expanded LM is steeper than the basic LM curve. This is because the basic LM curve captures only the impact of a rise in output on the transactions demand for money while the expanded LM captures this effect plus the portfolio effect arising from the output-induced rise (via saving) in total financial wealth.

\textsuperscript{15} Using saving is one approach to determining total financial wealth. An alternative is the “supply of assets” approach. This approach defines wealth in terms of the real supply of money and bonds and then links the flow supply of new assets to the spending decisions of firms and the government via their respective budget constraints. Specifically, \( a_t = a_0 + \Delta m^*_t + \Delta h^*_t + \Delta h^*_t \), where \( a_0 \) is the total real supply of money \((m_0)\) and bonds \((b_0)\) from the previous period, \( \Delta m^*_t \) is the current-period change in the real money supply, and \( \Delta h^*_t + \Delta h^*_t \) is the current-period change in the total real supply of bonds. While the change in the money supply is exogenous, the change in the real supply of government bonds is determined by the government budget constraint, \( \Delta h^*_t \equiv g - t - \Delta m^*_t \), while the change in the supply of private bonds is determined by the firm budget constraint, \( \Delta h^*_t \equiv i(t) \). Substituting from the budget constraints for \( \Delta h^*_t \) and \( \Delta h^*_t \) into the expression for \( a_t \) given above yields \( a_t = a_0 + (g-t) + i(t) \).
This is illustrated in Figure 3 which pairs the two LM curves (LM$_1$ being the basic LM curve and LM$_2$ the expanded curve) with a single IS curve. Now suppose that, starting from point A where the money market is in equilibrium on both curves, there is a $1$ rise in output to $y_2 = y_1 + 1$ while the interest rate remains unchanged at $r_1$. This corresponds to the movement from point A to point B. With LM$_1$, the rise in income increases the transactions demand for money by $m^d_y$, creating excess money demand equal to $m^d_y$ at point B. With LM$_2$, however, the increase in income: (1) increases the transactions demand for money by $m^d_y$ and (2) increases saving and hence total financial wealth by $(1-c_y)$ which increases money demand by $m^d_a(l-c_y)$. For LM$_2$, then, the excess demand for money at point B is equal to $m^d_y + m^d_a(l-c_y)$, which clearly is greater than the excess money demand for LM$_1$ at point B. As a result, it takes a larger rise in $r$ to restore equilibrium at $y_2$ on LM$_2$ ($r_3$ at point D) than on LM$_1$ ($r_2$ at point C). Thus, the expanded LM must be steeper.

Since $s = (g-t) + i(r)$, it follows that the two approaches to determining total real financial wealth are equivalent.

16 Again, for simplicity, we ignore any interest-induced wealth effects that would affect the slope of the LM curve.
V. Fiscal Policy Revisited

Having respecified the IS-LM model in such a way as to capture the impact on financial markets of increased bond supplies, we now revisit the macroeconomic consequences of a bond-financed rise in \( g \). Figure 4 pairs the IS curve with both LM curves from Figure 3. LM\(_1\) is the basic curve, and LM\(_2\) is the expanded curve. The initial short-run equilibrium is at point A where output is \( y_1 \) and the interest rate is \( r_1 \). Now suppose that there is a bond-financed rise in government expenditures from \( g_1 \) to \( g_2 \) that shifts the IS curve to IS\((g_2)\). Using the basic LM curve, the new short-run equilibrium will be at point B. Output increases to \( y_3 \) as the interest rate rises to \( r_2 \) via the output effect discussed earlier. Using the expanded LM curve, however, the demand for money also rises because of the increase in total real financial wealth, creating additional excess demand in the money market (as well as additional excess supply in the bond market). The portfolio effect, then, raises the interest rate beyond \( r_2 \) to \( r_3 \) (reducing the rise in output to \( y_2 \) from \( y_3 \)) as the economy moves to a short-run equilibrium at point C.

![Figure 4](image-url)
Intuitively, the increased supply of bonds required to finance the higher level of $g$ is no longer being offset by an equal increase in bond demand. Consequently, the bond demand curve shifts right by less the bond supply curve, putting additional upward pressure on the interest rate. We may properly call this a “bond supply effect,” and it is this effect that shows up in Figure 4 as the added rise in the interest rate from $r_2$ to $r_3$.

Mathematically, the multiplier for a bond-financed rise in government expenditures in the expanded IS-LM model is:

$$
\frac{dy}{dg} = \frac{1}{(1-c_y) + m_y^d \left( \frac{1}{m_r^d} \right) + m_a^d (1-c_y) \left( \frac{1}{m_r^d} \right)} = \frac{+}{+} > 0. \quad (15)
$$

The “portfolio” or “bond supply effect” is the third term in the denominator of equation 15, and it reduces (but does not eliminate) the increase in output in Figure 4. While the portfolio effect does not eliminate the increase in output in the current period (the current-period multiplier, as given by equation 15, is still positive), it will cause output to decline in future periods if the bond-financed rise in $g$ is permanent. To understand why this is so, consider what happens in period $t+1$. Government spending is unchanged at $g_2$ which means the IS curve remains stationary. Such is not the case, however, for the (expanded) LM curve. Period $t+1$ saving increases total financial wealth and therefore the demand for assets (i.e., the demand for money plus the demand for bonds).

Investment, $i(r)$, plus the government deficit ($g_2-t_1$) determines the period $t+1$ increase in the supply of bonds. And since saving must equal investment plus the government deficit, it follows that:

$$
s_{t,1} = \Delta m^d + \Delta b^d = \Delta a = \Delta b^s = i(r) + (g_2 - t_1). \quad (16)
$$
With $m^d > 0$, the increase in bond demand will be less than the increase in bond supply. This puts further upward pressure on the interest rate in $t+1$, crowds out additional investment spending, and causes output to fall. Graphically, this effect is captured by an upward shift in the LM curve in period $t+1$. This shift continues period after period so long as saving, hence the change in total financial wealth, is positive.

Mathematically, the period $t+1$ change in output resulting from the upward shift in the LM curve is:

$$\frac{dy}{da} = \frac{-m^d}{(1-c)(m^d/m_r + m^d/i_r) + m^d} < 0. \quad (17)$$

Equation 17 is unambiguously negative.\(^{18}\) So, following a bond-financed rise in $g$ that increases output in the current period (equation 15), output falls in future periods due to the continued upward pressure on the interest rate (and continued crowding out of private investment) caused by the increased supply of government bonds to finance the higher deficit.\(^{19}\)

If the bond-financed rise in $g$ causes output to rise in the first period and then fall in subsequent periods, the question that naturally arises is this: what is the long-run equilibrium level of $y$? Since saving increases total financial wealth and increases in total

\(^{17}\) If $m^d = 0$, equation 15 reduces to the basic IS-LM multiplier for a bond-financed rise in $g$. (See footnote 4).

\(^{18}\) To determine the period $t+1$ impact on output, we rewrote equations 10, 11, and 12 in linear form and solved for the reduced-form equation for output. When $m^d > 0$, total financial wealth enters this equation with a one-period lag and a negative effect on output. Since saving increments wealth, output must continue to fall so long as saving is positive. This effect (for one period) is given by equation 17.

\(^{19}\) Our analysis is contingent upon the absence of a wealth effect in the consumption function. If government bonds are considered wealth by the private sector, and if wealth has a positive effect on consumption, the IS curve would shift upwards in period $t+1$, changing our results. We ignore the wealth effect on consumption for two reasons. One, it has been discussed widely in the literature, and two, it detracts from our primary objective which is to explore the financial market impact of an increased supply of bonds.
financial wealth boost money demand \((m^d_s > 0)\), the LM curve will continue to shift upwards, causing \(r\) to rise and \(y\) to fall, as long as saving is positive. Saving must, therefore, be 0 in the long run. And since \(s = i(r) + (g - t)\), it follows that in long-run equilibrium:

\[
i(r) = -(g - t)
\]  

(18)

In other words, the long-run equilibrium level of output occurs where investment is negative and equal (in absolute value) to the government deficit!

To better understand this result, assume that the economy begins from a position of long-run equilibrium where \(s = 0\) [and where \(i(r) = -(g - t)\)]. Now suppose that there is a bond-financed rise in \(g\) that shifts the IS to the right causing output to rise above its long-run equilibrium level. Accompanying this rise in output is a rise in saving so that saving is now positive. As long as \(s > 0\), the LM curve will shift up in subsequent periods causing \(r\) to rise and \(y\) to fall. With each decline in \(y\) (and rise in \(r\)), there will be a decrease in saving (and investment). This process continues until investment is completely crowded by the rise in \(g\), that is, until \(\Delta i = -\Delta g\). At this point saving will, once again, be equal to zero, and output will have returned to its initial level. This necessarily implies that the long-run multiplier for a bond-financed rise in \(g\) equals 0.20

VI. Some Thoughts on Ricardian Equivalence and the Basic IS-LM Model

We conclude by noting that the basic IS-LM model implicitly incorporates one of the main results of the Ricardian Equivalence Theorem (RET)—namely that an increased

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20 Notice that this long-run neutrality of output with respect to a permanent increase in \(g\) financed by bond sales does not hinge on price flexibility as is the case in the basic IS-LM model.
supply of government bonds to finance an increase in \( g \) has no independent effect on the interest rate. This observation might be both surprising and disturbing to those who teach the basic model but do not endorse the RET. Of course, it does not follow that the basic model and the RET are \textit{wholly} consistent. Consider, for example, a tax cut financed by bond sales. According to the RET, current taxes and bond sales in lieu of current taxes have equivalent effects on the after-tax permanent income of households (since permanent income is defined to include the discounted value of future tax liabilities). Therefore, a tax cut financed by bond sales should have no effect on aggregate demand. In the basic model, however, a tax cut does raise aggregate demand because consumption depends only on current disposable income, which is rising, and not on permanent income, which takes future tax increases into account.

There are two conditions that are sufficient to render the IS-LM model and the RET fully compatible. These conditions are:

1. Total real financial wealth must not appear as an argument in the money demand function.

2. The “income” variable in the consumption function is permanent or lifetime income that takes into account the discounted value of all future tax liabilities.

Condition 1, we have argued, amounts to assuming away the existence of an independent effect on the interest rate of government bond sales to finance a rise in \( g \) (or, for that matter, a cut in taxes). Most textbooks implicitly impose this condition without recognizing its implications. Condition 2 insures that consumption is determined independently of government bond holdings and also that taxes and bond sales are viewed as equivalent operations by the private sector. A tax cut financed by bond sales
will not raise consumption, and it will not matter whether the government finances a
given level of spending through taxes or bond sales.

Aside from Barro (2008) and Williamson (2008), few texts contain more than a
cursory discussion of the RET. And those texts that do go into detail claim that condition
2 is, by itself, sufficient for the RET to hold in the basic IS-LM model. This appears to be
because these texts (implicitly) assume that condition 1 holds. Consequently, authors of
many leading macroeconomic texts have failed to recognize, if only inadvertently, the
importance of condition 1 in generating results that are consistent with the RET.

VII. Conclusion

In this paper we have discussed how the interest rate effects of increased bond
supplies to finance an increase in government spending are typically modeled. From a
pedagogical perspective, we have found much that teachers in this area should find
confusing. The basic IS-LM model that is commonly employed assumes there is no
financial market impact on the interest rate from the sale of government bond required to
finance a higher level of government spending. Instead, it is assumed that an increased
supply of government bonds is automatically absorbed into private sector portfolios. As
a result, any deficit resulting from an increase in government expenditures has no
independent effect on the economy. In other words, deficits do not matter. When we
expand the IS-LM model to capture the impact on interest rates from an increased supply
of bonds, we find that deficits do matter, and that they matter in a way consistent with the
heightened concerns expressed by professional economists and the financial community.
References


