Is Tax Sharing Optimal?
An Analysis in a Principal-Agent Framework

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ABSTRACT. We study the effects of a statutory wage tax sharing rule in a principal-agent framework with moral hazard (à la Holmstrom, 1979) The analysis indicates that tax sharing with positive legislated contributions from both the employer and employee does not maximize any of the relevant outcomes – employee effort, wages, profits or welfare. Moreover, a rule which specifies a corner solution, with 100% of the tax statutorily levied on the employer will maximize expected profit and expected welfare while 100% of the tax statutorily levied on the employee will maximize effort and expected wages.

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1. INTRODUCTION

During the past three decades, the principal-agent framework has become an integral part of economic modeling. (See Sappington, 1991, and Laffont and Martimort, 2002). When the principal cannot observe the agent’s effort, she designs an optimal compensation schedule to induce optimal effort. Frequently, policies enforced by a third party have substantial impact on the optimal contract. The common practice of wage taxation is one such policy.

The taxation of wage income in various forms, is common throughout the world. In approximately half of all OECD countries, the shares of employer/employee contributions toward a social security tax, for example, has been stable at approximately 25% of total labor costs. Yet, the distribution of this share between employer/employee varies widely across countries. There is a 50:50 split in Germany, Switzerland, United States, Luxembourg and Japan. In most other countries, employers typically pay the major share. The exceptions are Denmark and the Netherlands, where employees generally pay the most. This variation and the lack of formal analysis in the literature, motivate the present study.

The question investigated in this paper is the effect of a statutory wage tax sharing rule on wages, effort, profits and aggregate welfare. In a typical complete information competitive model, the standard result of the neutrality of tax shares is well known. However, incomplete information and imperfect competition are more often the rule, rather than exceptions. In that context, the effect of tax sharing rules and any analysis on the optimality of these rules, is not especially evident in the literature. Our model is a principal (employer) - agent (employee) framework with moral hazard (à la Holmstrom, 1979) where an employee’s effort is unobservable by his employer and at the same time, is affected by the

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taxes imposed on his wage income. The tax sharing rule is set ex ante by the government—prior to any decisions made by either the principal or the agent.

Our results show that tax sharing with positive legislated contributions from both the employer and employee, does not maximize any of the relevant outcomes: effort, wages, profits or welfare. Moreover, it turns out that a statutory sharing rule which specifies that 100% of the tax statutorily levied on the employer will, under some conditions, maximize expected profit and welfare while 100% of the tax statutorily levied on the employee will, under some conditions, maximize effort and expected wages.

There is a substantial literature on the optimal income tax in adverse selection models (see Diamond, 1998 and Seade, 1977). The focus of our work is on moral hazard and risk sharing. In the theoretical literature studying the impact of taxes on hours of work, the typical conflict between the substitution effect and income effect has rendered any conclusion logically indeterminate.\(^1\) While this paper is related to the topic of taxation under uncertainty (see Eaton and Rosen 1980 a, b; Rosen, 1980), it is fundamentally different in the model being used and the implication of a statutory tax sharing rule that is being studied. It is useful to see the work by Feldstein (1995) for discussion on the extent to which taxable income as a whole, and not just labor supply, responds to changes in marginal tax rates.

The next section describes the model. The analysis with fixed reservation utility is analyzed in Section 3 while Section 4 allows reservation utility to be a function of the tax sharing parameters. Section 5 concludes the paper.

2. MODEL

The basic framework is the familiar one of a risk neutral employer (or principal) and risk averse employee (or agent) who works for wages, which are taxed by the government. Further, a statutory distribution of the tax between the employer and employee, is mandated by the government. The employer-employee relationship involves moral hazard, where the agent’s effort is unobservable by the principal, but it affects the expected outcome as well as the riskiness of outcomes. The realized output is a noisy signal of the agent’s effort. Therefore the principal wants to use the contract to induce the agent to exert optimal effort.

Let \(a\) denote agent’s effort, \(x \in [0, \infty)\) be realized output level, with conditional density function \(f(x|a)\). Let \(w(x)\) be payment from the principal to the agent when output \(x\) is realized. Denote by \(U_A(w, a) = 2(w) - a^2\) the agent’s utility function. We set \(\eta = 1/2\) to satisfy Jewitt (1988) conditions, so the first order approach can be used.\(^2\) Let \(U\) be his reservation utility level. The principal’s payoff function is given by \(U_P(x, w) = \pi(x, w) = x - w\). The government sets a tax rate on wage \(w(x)\), denoted by \(t\). Employees pay a fraction \(\gamma\), of this wage tax. Hence welfare, \(W(x) = x - (1 - t\gamma)w(x) + 2(1 - t\gamma)^{1/2}w(x)^{1/2} - a^2\).

We use the gamma distribution in this model, for \(f(x|a)\). Its flexibility and general properties are well suited for use in this framework. See Bose, Pal and Sappington, 2007 for this idea and justification for using the gamma distribution in a principal-agent framework.

Note that the density function for the gamma distribution is given by:

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\(^1\)Eaton and Rosen (1980 a) summarize the extensive econometric research as suggestive of very small responses in hours of work to changes in net wage for prime male earners. However, other groups, such as married women, have considerably higher labor supply response rates.

\(^2\)Note that \(\eta \leq \frac{1}{2}\), will satisfy Jewitt’s conditions. We use \(\eta = 1/2\), to get explicit solutions.
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\[ f(x|a) = f(x; p, a) = \frac{1}{a^p \Gamma(p)} x^{(p)-1} e^{-x/a}, \text{ for } x \in [0, \infty) \]

where

\[ \Gamma(p) = \int_0^\infty e^{-x} x^{(p)-1} dx. \]

3. CONSTANT RESERVATION UTILITY

We will first consider the standard principal agent framework with moral hazard, with constant reservation utility.

The Principal’s problem \([P]\) can be written as:

Maximize \(w; a \int_0^\infty [x - w(x) - (1 - \gamma)tw(x)]f(x|a)dx\)

subject to the Participation Constraint

\[ \int_0^\infty 2(1 - tw(x))f(x|a)dx - a^2 = \overline{U}, \]

and the Incentive Compatibility Constraint

\[ \int_0^\infty 2(1 - tw(x))f(x|a)dx - 2a = 0. \]

The solution to \([P]\) is characterized in Theorem 1. We assume that \(\overline{U}\) is sufficiently large such that \(w'(x) > 0\) for all \(x \geq 0\).

Theorem 1. The solution to the Principal’s Problem \([P]\) (second best solution) is characterized by the following set of equations:

\[ \lambda = \frac{1 + t - t\gamma}{2(1 - t\gamma)}(a^2 + \overline{U}), \]

\[ \mu = \frac{(1 + t - t\gamma)a^3}{p(1 - t\gamma)}, \]

\[ a = \sqrt[3]{-L + \sqrt{L^2 + K^3}} + \sqrt[3]{-L - \sqrt{L^2 + K^3}}, \]

\[ w(x) = \frac{1}{4(1 - t\gamma)} \left( \frac{2a}{p} x + \overline{U} - a^2 \right)^2, \]

where

\[ K := \frac{p\overline{U}}{3(p + 4)} \quad \text{and} \quad L := -\frac{p^2(1 - t\gamma)}{2(p + 4)(1 + t - t\gamma)}. \]

Proof. See Appendix 1.

Next we investigate the relationship between the employee’s tax share \(\gamma\) and his optimal effort and expected wage, the principal’s profit and the aggregate welfare. These results are presented in Theorem 2.

Theorem 2. The following relations hold for all \(\gamma \in [0, 1]\):

\(a\) \(\frac{\partial \mu}{\partial \gamma} < 0;\)

\(b\) If \(1 - t > (2 + \gamma)t\), then \(\frac{\partial E(w)}{\partial \gamma} > 0;\)
(c) $\frac{\partial E(\pi)}{\partial \gamma} < 0$;

(d) $\frac{\partial E(W)}{\partial \gamma} < 0$.

where $E(w)$ is expected wage, $E(\pi)$ is expected profit and $E(W)$ is expected welfare.

Proof. See Appendix 2.

From Theorem 2(a), we see that the agent’s optimal effort is a strictly declining function of his share of the wage tax. The higher his mandated share of wages that are taxed, the lower his effort. If the share of net wages $(1 - t)$ is at least three times the taxed share $t$, then the expected wage increases with the share of the tax paid by the employee, $\gamma$, as stated in Theorem 2(b). Computation results in the next section show that indeed this result is always true. Intuitively, in the face of uncertainty with respect to output and hence, wages, the principal (or employer) has to compensate the risk averse agent (or employee), in the event of higher $\gamma$, with a higher expected wage. For this same reason, the principal’s expected profit falls with $\gamma$, as shown in Theorem 2(c). It turns out that expected welfare is a strictly decreasing function of the agent’s share of the wage tax, as stated in Theorem 2(d). These results imply that while expected wage is maximized if the agent is legislatively mandated to pay 100% of the wage tax, the agent’s effort, expected profit and expected welfare are all minimized under that tax rule. What is clear is that regardless of the tax distribution chosen as a policy matter, an interior solution with positive shares for both principal and agent, does not optimize any of these outcomes.

The numerical computation below, further explores our theoretical findings in Theorem 2.

3.1. Computation results.

Conclusion 1. Employee effort $a$, expected firm profit $E(\pi)$ and expected welfare $E(W)$ are maximized when the statutorily mandated employee’s share of the tax $\gamma$ is zero; employee expected wage $E(w)$ is maximized when the statutorily mandated employee’s share of the tax is one.

Tables 1 - 4 report the numerical results for $p = 3$, $U = 1.5$, $t$ and $\gamma$ varying from 0.1 to 0.9 in increments of 0.1. The results corresponding to values of $t$, $\gamma$, $p$ and $U_0$ satisfying the sufficient conditions in Theorem 2 are highlighted in bold. Consistent with Theorem 2(a), given $t$, employee effort decreases with an increase in $\gamma$ (see Table 1). We also see that given $t$, the expected wage increases with $\gamma$. Note that this monotonic relationship is stronger than stated in the sufficient condition in Theorem 2(b) (see Table 2). As stated in Theorem 2(c), computation results verify that expected profit decreases with $\gamma$ for given $t$ (see Table 3). Simulation results in Table 4 support the result in Theorem 2(d), that expected welfare falls with $\gamma$, given $t$. 
4. RESERVATION UTILITY IS A FUNCTION OF TAX REGIME

We next extend the standard moral hazard framework to further explore the implications of the tax regime. We argue that the agent’s willingness to accept a contract will vary depending on the tax environment. In other words, the agent’s opportunity wage might be affected by the prevailing tax rate. The question is to what extent this will qualitatively affect the results from the previous section.

Define the agent’s reservation utility as:

\[ U = U_0 (1 - t \gamma)^\theta, \]

for \( \theta > 0 \). Hence, \( \theta = \frac{\partial U}{\partial (1 - t \gamma)} \), the elasticity of the agent’s utility \( U \) with respect to his mandated (post) tax share \( (1 - t \gamma) \). Therefore, this specification of the reservation utility will permit the analysis of the responsiveness of the relevant variables, effort, profits, wages and welfare, conditional on this elasticity, \( \theta \).

The Principal’s problem \([P1]\) can now be written as:

Maximize \( w, a \int_0^\infty [x - w(x) - (1 - \gamma)t w(x)] f(x | a) dx \)

subject to

\( \int_0^\infty 2 \sqrt{w(x) - t \gamma w(x)} f(x | a) dx - a^2 = U_0 (1 - t \gamma)^\theta \)

and

\( \int_0^\infty 2 \sqrt{w(x) - t \gamma w(x)} f_a(x | a) dx - 2 a = 0. \)

Note that the results in Theorem 1 hold as the solution to problem \([P1]\).

The effects of the employee’s share of the tax on effort, expected wages, profits and welfare, are summarized in Theorem 3.

**Theorem 3.** (a) If \( \frac{\partial U}{\partial (1 - t \gamma)} < \frac{pt}{(1 + t - t \gamma)^2} \), then \( \frac{\partial a}{\partial \gamma} < 0 \);

(b) If \( 0 < \theta \leq 1/2 \) and \( 1 - t > (2 + \gamma)t \), then \( \frac{\partial E(w)}{\partial \gamma} > 0 \);

(c) If \( 0 < \theta \leq 1/2 \), then \( \frac{\partial E(\pi)}{\partial \gamma} < 0 \);

(d) If \( \frac{\partial U}{\partial (1 - t \gamma)} < \frac{pt}{(1 + t - t \gamma)^2} \) and \( 0 < \frac{p(1 - t \gamma)}{\sqrt{2(1 + t - t \gamma)}} < U < 2 \) then \( \frac{\partial E(W)}{\partial \gamma} < 0 \).

**Proof.** See Appendix 3.

In (a) above, we find that the conclusion from Theorem 2(a) holds, provided that the variation of \( U \) with respect to \( 1 - t \gamma \) is not "too high". Specifically, this variation depends on both \( t \) and \( \gamma \). If \( t \) is close to zero, then this variation is negative. Intuitively, this means that as net wage increases, the agent will settle for a lower reservation utility. If \( t \) is close to one, then this variation should not be higher than \( p/2 \), where \( p \) is the productivity of effort. In (b) we replicate the conclusion from Theorem 2(b), provided the elasticity \( \theta \leq 1/2 \).

This means that reservation utility must be somewhat inelastic with respect to net wages. In (c), we replicate the result from Theorem 2(c), for all \( 0 < \theta \leq 1/2 \). Provided that the elasticity is not "too high" and the reservation utility is between the bounds stated above, in
(d) we replicate the result in Theorem 2(d). Note that the bounds on $U$ are valid, provided $p < 4\sqrt{2}$, irrespective of values of $t$ and $\gamma$.

In conclusion, if the agent’s reservation utility is not "too responsive" to changes in the share of his wages that must, by legislation, be paid in taxes, the above results hold. This can be compared, somewhat loosely, to the inverse relationship between the elasticities of demand and supply and the actual tax burdens in the competitive model. There, the legislated tax burden has no impact on the actual tax burden; the latter is determined by the elasticities of demand and supply. Somewhat contrary to the conventional wisdom in that framework, here that result no longer holds. The legislated tax share has real implications for effort, expected wage, profit and expected welfare. This is strikingly different from the neutrality of the impact of tax mandates on actual tax burdens in the conventional full information competitive framework.

We next present some computation results which are summarized in the following conclusion below:

4.1. **Computation results.**

**Conclusion 2.** With variable reservation utility, for sufficiently small values of the elasticity parameter $\theta$ employee effort is maximized when the statutorily mandated employee’s share of the tax is zero; for sufficiently small values of $\theta$ and $t$ employee expected wage is minimized when the statutorily mandated employee’s share of the tax $\gamma$ is zero; expected profit is maximized at $\gamma = 0$, again with $\theta$ sufficiently small; expected welfare actually switches corners for its maximized value, as the parameter values change; however, within the range of the sufficient conditions stated in Theorem 3, expected welfare decreases monotonically with $\gamma$.

Numerical results with variable reservation utility are reported in Tables 5 - 8, for $U_0 = 1.5$, $p = 3$ and $\theta = 1/4$. (The results corresponding to values of $t, \gamma, p$ and $U_0$ satisfying the sufficient conditions in Theorem 3, are highlighted in bold.) The results in Table 5 correlate with the results in Table 1, for the range of parameter values that satisfy the sufficient conditions in Theorem 3(a); i.e. there is an inverse relationship between effort and employee’s tax share, given tax rate, $t$. Similarly, the results in Table 6, correspond to those in Table 2 for expected wage. In fact, simulation suggests that the increasing monotonic relationship between $E(w)$ and $\gamma$ is more robust than that suggested in Theorem 3(b). However, expected profit with variable reservation utility (Table 7) behaves somewhat differently than when reservation utility is fixed. In the latter case, it is always decreasing with $\gamma$. In the former case, that is not necessarily true. We observe that it is true for somewhat "higher" tax rates $t$ but in fact, switches from being maximized at high values of $\gamma$ with low tax rates $t$, to the opposite corner at low values of $\gamma$ for higher $t$. Table 8 shows that expected welfare decreases monotonically with $\gamma$, when sufficient conditions are satisfied, and sometimes otherwise. It actually increases for low values of tax rate $t$.

In the first best case, it can be shown relatively easily that (with fixed reservation utility) profit and welfare are maximized when the employee’s statutory tax share is 100%, while simultaneously wages and tax revenue are minimized. Clearly the moral hazard intrinsic in the second best case is critically important to the results obtained here.

5. **Conclusion**

In the presence of incomplete information, the statutory liability of a tax has unmistakable implications for profits, wages and aggregate welfare. This paper uses a principal-agent moral hazard framework to address this question. The theoretical and numerical
results do not find any justification for distributing the burden of a wage tax between employer and employee. The conclusions suggest that a careful examination exploring the connection between mandated tax liability, its implications for employer and employee earnings and what optimal policy should be in this context, warrants further study.

REFERENCES


6. APPENDIX 1: PROOF OF THEOREM 1

Recall the following: The gamma distribution has many general properties that are well suited for use in this framework. We recall in particular that

\[(6.1) \int (Ax + B)^2 f(x|a) \, dx = A^2a^2p(p + 1) + 2ABap + B^2 = A^2a^2p + (Aap + B)^2\]

for any constants A and B and that

\[(6.2) \frac{f_a(x|a)}{f(x|a)} = \frac{x - ap}{a^2},\]

where \(f_a\) is the partial derivative of \(f(x|a)\).

Write

\[(6.3) \alpha := 1 + (1 - \gamma)t \quad \text{and} \quad \beta := 2\sqrt{(1 - t\gamma)}\]

for brevity. Then the Principal’s Problem \([P]\) can be re-written as follows:

Maximize \(w, a \int_0^\infty [x - \alpha w(x)] f(x|a) \, dx\)

subject to

\(\int_0^\infty \beta \sqrt{w(x)} f(x|a) \, dx - a^2 = U\)

and

\(\int_0^\infty \beta \sqrt{w(x)} f_a(x|a) \, dx - 2a = 0.\)
The Lagrangian corresponding to the Principal’s Problem is:

\[ L = \int_0^\infty \left[ x - \alpha w(x) \right] f(x|a) dx + \lambda \left[ \int_0^\infty \beta \sqrt{w(x)} f(x|a) dx - a^2 - U \right] + \mu \left[ \int_0^\infty \beta \sqrt{w(x)} f_a(x|a) dx - 2a \right]. \]

Theorem 1 follows from the maximization of \( L \) with respect to \( w, \lambda, \mu \) and \( a \).

Step 1. Pointwise maximization of \( L \) with respect to \( w \) yields

\[
\frac{\partial L}{\partial w} = -\alpha f(x|a) + \frac{\lambda \beta}{2 \sqrt{w(x)}} f(x|a) + \frac{\mu \beta}{2 \sqrt{w(x)}} f_a(x|a) = 0
\]

Solving for \( w(x) \) using (6.2) gives

\[
(6.4) \quad w(x) = \left[ \frac{\beta}{2\alpha} \left( \lambda + \mu \frac{x - ap}{a^2} \right) \right]^2.
\]

Step 2. Maximization of \( L \) with respect to \( \lambda \), using (6.4) gives:

\[
\int_0^\infty \beta \sqrt{w(x)} f(x|a) dx - a^2 - U = 0
\]

\[
\frac{\beta^2}{2\alpha} \int_0^\infty \left[ \lambda + \mu \frac{x - ap}{a^2} \right] f(x|a) dx = a^2 + U
\]

Noting that

\[
(6.5) \quad \int_0^\infty \frac{x - ap}{a^2} f(x|a) dx = 0
\]

we have:

\[
(6.6) \quad \lambda = \frac{2\alpha}{\beta^2} \left( a^2 + U \right).
\]

where the last equality follows from (6.1).

Step 3. Maximization of \( L \) with respect to \( \mu \) yields:

\[
(6.7) \quad \int_0^\infty \beta \sqrt{w(x)} f_a(x|a) dx - 2a = 0.
\]

Using (6.2), (6.4) and (6.6), we have

\[
\frac{\mu \beta^2}{2\alpha a^4} \int_0^\infty (x - ap)^2 f(x|a) dx - 2a = 0,
\]

which gives using (6.1) and solving for \( \mu \)

\[
\mu = \frac{4\alpha a^3}{\beta^2p}.
\]
Finally, plugging the expression of $\mu$, $\lambda$, $\alpha$ and $\beta$ above gives the expression of the wage function (3.6).

Step 4. Maximization of $L$ with respect to $a$ yields:

\[
\int_0^\infty [x - \alpha w(x)] f_a(x|a) dx = \int_0^\infty \frac{x - ap}{a^2} f(x|a) dx + \mu \left[ \int_0^\infty \beta \sqrt{w(x)} f_{aa}(x|a) dx - 2 \right] = 0.
\]

where the term in $\lambda$ cancels out from (6.7) and

\[
\int_0^\infty [x - \alpha w(x)] f_a(x|a) dx = \int_0^\infty \frac{x - ap}{a^2} f(x|a) dx - \alpha \int_0^\infty \left\{ \frac{\beta^2}{4a^2} \left( \lambda^2 + 2\lambda \mu \left( \frac{f_a}{f} \right) \right) + \left( \frac{f_a}{f} \right)^2 \right\} f_a(x|a) dx
\]

\[
= \int_0^\infty \frac{x - ap}{a^2} f(x|a) dx - 2\alpha \lambda \mu \frac{\beta^2}{4a^2} \int_0^\infty \left( \frac{f_a}{f} \right) f_a(x|a) dx - \frac{\beta^2}{4a^2} \mu^2 \int_0^\infty \left( \frac{f_a}{f} \right)^2 f_a(x|a) dx
\]

\[
= \int_0^\infty \left[ \frac{(x - ap)(x - ap)}{a^2} \right] f(x|a) dx - \frac{\lambda \mu \beta^2}{2a} \int_0^\infty \frac{x - ap}{a^2} f(x|a) dx - \frac{\beta^2 \mu^2}{4a} \int_0^\infty \left( \frac{x - ap}{a^2} \right)^2 f(x|a) dx
\]

\[
= p - \frac{\lambda \mu \beta^2 p}{2a^2} - \frac{\beta^2 \mu^2 p}{2a^2}.
\]

and

\[
\int_0^\infty \beta \sqrt{w(x)} f_{aa}(x|a) dx
\]

\[
= \int_0^\infty \frac{\beta^2}{2a} \left( \lambda + \mu \frac{f_a}{f} \right) \left( \frac{(x - ap)^2}{a^4} + \frac{ap - 2x}{a^3} \right) f(x|a)
\]

\[
= \frac{\beta^2 \lambda p}{2a^2} + \frac{\beta^2 \mu p}{a^3} + \frac{\beta^2 \lambda p}{2a^2} \int_0^\infty \frac{x - ap}{a^2} \cdot \frac{(x - ap)^2}{a^4} f(x|a) dx
\]

\[
+ \int_0^\infty \frac{\beta^2}{2a} \left( \lambda + \mu \frac{f_a}{f} \right) \frac{ap - 2x}{a^3} f(x|a) dx
\]

\[
= \frac{\beta^2 \lambda p}{2a^2} + \frac{\beta^2 \mu p}{a^3} + \frac{\beta^2 \lambda p}{2a^2} \int_0^\infty \frac{x - ap}{a^2} \cdot \frac{(x - ap)^2}{a^4} f(x|a) dx
\]

\[
+ \int_0^\infty \frac{\beta^2 \mu}{2a} \cdot \frac{x - ap}{a^2} \cdot \frac{ap - x}{a^3} f(x|a) dx
\]

\[
+ \int_0^\infty \frac{\beta^2}{2a} \cdot \frac{x - ap}{a^2} \cdot \frac{-x}{a^3} f(x|a) dx
\]

\[
= \frac{\beta^2 \lambda p}{2a^2} + \frac{\beta^2 \mu p}{a^3} - \frac{\beta^2 \lambda p}{2a^2} - \frac{\beta^2 \mu p}{a^3}.
\]
Using the above results in equation (6.8) we get
\[ p - \frac{\lambda \mu^2 p}{2\alpha a^2} - \frac{\beta^2 \mu^2 p}{2\alpha a^3} - 2\mu = 0 \]
or
\[ (6.9) \quad p - 2\lambda a - \frac{16\alpha a^3}{\beta^2 p} = 0 \]
where the last equality is obtained by using the expression of \( \mu \).

Multiplying (3.3) by \( 2a \) and adding it to (6.9) we eliminate \( \lambda \) and we obtain a cubic equation for \( a \):
\[ \frac{p + 4}{p} a^3 + \frac{4}{a} - \frac{\beta^2 p}{4\alpha} = 0. \]

Using (6.3) we may rewrite it as
\[ (6.10) \quad (p + 4)a^3 + pUa = \frac{p^2(1 - t\gamma)}{1 + t - t\gamma} \]
and using (3.7) also as
\[ a^3 + 3Ka + 2L = 0. \]

Since the discriminant
\[ D := -108(K^3 + L^2) \]
of this equation is (strictly) negative, our equation has one real and two complex roots. Moreover, since \( L < 0 \), the real root is (strictly) positive. Finally, this positive root is given by the Tartaglia–Cardano formula (3.5).

**APPENDIX 2: PROOF OF THEOREM 2**

*Proof of (a): \( \partial a / \partial \gamma < 0 \).* We deduce from (6.10) that
\[ \frac{\partial a^3}{\partial \gamma} = \frac{p^2}{p + 4} \cdot \frac{-t^2}{(1 + t - t\gamma)^2} - \frac{p}{p + 4} \frac{\partial a}{\partial \gamma}. \]

Since
\[ \frac{\partial a^3}{\partial \gamma} = 3a^2 \frac{\partial a}{\partial \gamma}, \]
it follows that
\[ \frac{\partial a}{\partial \gamma} = \frac{-t^2 p^2}{(1 + t - t\gamma)^2(pU + 3(p + 4)a^2)} < 0. \]
\( \square \)

*Proof of (b): \( \partial E(w) / \partial \gamma > 0 \).* Using the equality (6.1) we have
\[
E(w) = \int w(x)f(x|a)\,dx \\
= \frac{1}{4(1 - t\gamma)} \int \left[ \frac{2a}{p} x + U - a^2 \right]^2 f(x|a)\,dx \\
= \frac{1}{4(1 - t\gamma)^{-1}} \left[ \left( \frac{2a}{p} \right)^2 a^2 p + \left( \frac{2a}{p} ap + U - a^2 \right)^2 \right] \\
= \frac{1}{4(1 - t\gamma)^{-1}} \left[ \frac{4a^4}{p} + 4a^4 + 4a^2(U - a^2) + (U - a^2)^2 \right],
\]
so that

\[ E(w) = \frac{1}{4p}(1 - t\gamma)^{-1} \left[ (p + 4)a^4 + 2p\bar{U}a^2 + p(\bar{U})^2 \right]. \]

Hence

\[
\frac{\partial E(w)}{\partial \gamma} = \frac{t}{4p(1 - t\gamma)^2} \left[ (p + 4)a^4 + 2p\bar{U}a^2 + p(\bar{U})^2 \right] \\
+ \frac{1}{4p(1 - t\gamma)} \left[ 4(p + 4)a^3 + 4p\bar{U}a \right] \frac{\partial a}{\partial \gamma}
\]

\[
= \frac{t}{4p(1 - t\gamma)^2} \left[ (p + 4)a^4 + 2p\bar{U}a^2 + p(\bar{U})^2 \right] \\
+ \frac{1}{p(1 - t\gamma)} \frac{p^2(1 - t\gamma)}{1 + t - t\gamma} \cdot \frac{-t^2p^2}{(1 + t - t\gamma)^2[p\bar{U} + 3(p + 4)a^2]}
\]

\[
= \frac{t}{4p(1 - t\gamma)^2} \left[ (p + 4)a^4 + 2p\bar{U}a^2 + p(\bar{U})^2 \right]
- \frac{p^3t^2}{(1 + t - t\gamma)^3[p\bar{U} + 3(p + 4)a^2]}
\]

It follows that

\[
\frac{\partial E(w)}{\partial \gamma} > \frac{t}{4p(1 - t\gamma)^2} \left[ (p + 4)a^4 + p\bar{U}a^2 \right] - \frac{p^3t^2}{(1 + t - t\gamma)^3[p\bar{U} + (p + 4)a^2]}
\]

\[
= \frac{at}{4p(1 - t\gamma)^2} \cdot \frac{p^2(1 - t\gamma)}{1 + t - t\gamma} - \frac{ap^3t^2}{(1 + t - t\gamma)^3} \cdot \frac{1 + t - t\gamma}{p^2(1 - t\gamma)}
\]

\[
= \frac{apt(1 - 3t - t\gamma)}{4(1 - t\gamma)(1 + t - t\gamma)^2}.
\]

We conclude that \( \frac{\partial E(w)}{\partial \gamma} \) has the same sign as \( 1 - 3t - t\gamma > 0 \).

\[
\text{Proof of (c): } \frac{\partial E(\pi)}{\partial \gamma} < 0. \text{ Differentiating the equality}
\]

\[
E(\pi) = \int [x - (1 + t - t\gamma)w(x)]f(x|a)\,dx = ap - (1 + t - t\gamma)E(w)
\]
we obtain that

\[
\frac{\partial E(\pi)}{\partial \gamma} = p \frac{\partial a}{\partial \gamma} + t E(w) - (1 + t - t\gamma) \frac{\partial E(w)}{\partial \gamma} = \frac{-p^2 t^2}{(1 + t - t\gamma)^2 [p\bar{U} + 3(p + 4)a^2]}
\]

\[
+ \frac{t}{4p(1 - t\gamma)} [(p + 4)a^4 + 2p\bar{U}a^2 + p(\bar{U})^2]
\]

\[
- \frac{t(1 + t - t\gamma)}{4p(1 - t\gamma)^2} [(p + 4)a^4 + 2p\bar{U}a^2 + p(\bar{U})^2]
\]

\[
+ \frac{p^2 t^2}{(1 + t - t\gamma)^2 [p\bar{U} + 3(p + 4)a^2]}
\]

\[
= \frac{-t^2}{4p(1 - t\gamma)^2} [(p + 4)a^4 + 2p\bar{U}a^2 + p(\bar{U})^2] < 0.
\]

Proof of (d): \( \partial E(W)/\partial \gamma < 0 \). Since

\[
W(x) = x - (1 - t\gamma)w(x) + 2(1 - t\gamma)^{1/2}w(x)^{1/2} - a^2,
\]

we have

\[
E(W) = \int W(x) f(x|a) \, dx = ap - (1 - t\gamma)E(w) + \int \left(\frac{2ax}{p} - a^2 + \bar{U}\right) f(x|a) \, dx - a^2.
\]

In the last integral the absolute value can be omitted if \( \bar{U} \geq a^2 \). In view of (6.10) this is equivalent to the inequality

\[
p(\bar{U})^{3/2} + (p + 4)(\bar{U})^{3/2} \geq \frac{p^2(1 - t\gamma)}{1 + t - t\gamma}
\]

which is assumed in the theorem. Under this assumption we have

\[
E(W) = ap - a^2 - (1 - t\gamma)E(w) + \int \left(\frac{2ax}{p} - a^2 + \bar{U}\right) f(x|a) \, dx
\]

\[
= ap - a^2 - (1 - t\gamma)E(w) + \frac{2a}{p} ap - a^2 + \bar{U}.
\]

Using (6.12) it follows that

(6.13)

\[
E(W) = E(\pi) + \bar{U} + tE(w).
\]
Differentiating with respect to $\gamma$ and using the expression of $\partial E(\pi)/\partial \gamma$ and $\partial E(w)/\partial \gamma$ obtained above, we obtain that

$$\frac{\partial E(W)}{\partial \gamma} = \frac{\partial E(\pi)}{\partial \gamma} + t \frac{\partial E(w)}{\partial \gamma}$$

$$= \frac{-t^2}{4p(1 - t\gamma)^2} \left[ (p + 4)a^4 + 2pUa^2 + p(U)^2 \right]$$
$$+ \frac{t^2}{4p(1 - t\gamma)^2} \left[ (p + 4)a^4 + 2pUa^2 + p(U)^2 \right]$$
$$- \frac{pt^3}{(1 + t - t\gamma)^3[pU + 3(p + 4)a^2]}$$

$$= \frac{-pt^3}{(1 + t - t\gamma)^3[pU + 3(p + 4)a^2]} < 0.$$ 

\[\square\]

**APPENDIX 3: PROOF OF THEOREM 3**

*Proof of (a):* \(\partial a/\partial \gamma < 0\). Differentiating (6.10) we get

$$[3(p + 4)a^2 + pU] \frac{\partial a}{\partial \gamma} + p \frac{\partial U}{\partial \gamma} = \frac{-p^2t^2}{(1 + t - t\gamma)^2}.$$ 

Since

$$\frac{\partial U}{\partial \gamma} = t\theta U_0(1 - t\gamma)^{\theta - 1} = \frac{-t\theta U}{1 - t\gamma},$$

it follows that

$$\frac{\partial a}{\partial \gamma} = \frac{pt}{3(p + 4)a^2 + pU} \left( \frac{\theta U}{1 - t\gamma} - \frac{pt}{(1 + t - t\gamma)^2} \right)$$
$$= \frac{pt}{3(p + 4)a^2 + pU} \left( \frac{\theta U_0}{(1 - t\gamma)^{1 - \theta}} - \frac{pt}{(1 + t - t\gamma)^2} \right).$$

It follows that the sign of \(\partial a/\partial \gamma\) is the same as that of

$$\theta U_0(1 - t\gamma)^{\theta - 1} - \frac{pt}{(1 + t - t\gamma)^2}.$$ 

\[\square\]
Proof of (b): \( \partial E(w)/\partial \gamma > 0 \). Using (6.10), (6.11) and the expressions of \( \partial a/\partial \gamma \) and \( \partial U/\partial \gamma \) obtained above, we get

\[
\begin{align*}
\frac{\partial E(w)}{\partial \gamma} &= \frac{t}{4p(1-t\gamma)^2} \left[ (p+4)a^4 + 2pUa^2 + p(U)^2 \right] \\
&\quad + \frac{1}{4p(1-t\gamma)} \left[ 4(p+4)a^3 + 4pUa \right] \frac{\partial a}{\partial \gamma} \\
&\quad + \frac{1}{4p(1-t\gamma)} \left[ 2pa^2 + 2pU \right] \frac{\partial U}{\partial \gamma} \\
&= \frac{t}{4p(1-t\gamma)^2} \left[ (p+4)a^4 + 2pUa^2 + p(U)^2 \right] \\
&\quad + \frac{p^2t}{(1+t-t\gamma)[3(p+4)a^2 + pU]} \left( \frac{\theta U}{1-t\gamma} - \frac{pt}{(1+t-t\gamma)^2} \right) \\
&\quad - \frac{(a^2 + U)t\theta U}{2(1-t\gamma)^2}.
\end{align*}
\]

Since

\[
\frac{p^2(1-t\gamma)}{a(1+t-t\gamma)} < 3(p+4)a^2 + pU < \frac{3p^2(1-t\gamma)}{a(1+t-t\gamma)}
\]

by (6.10), it follows that

\[
\frac{\partial E(w)}{\partial \gamma} > \frac{t}{4p(1-t\gamma)^2} \left[ (p+4)a^4 + 2pUa^2 + p(U)^2 \right] \\
&\quad + \frac{at\theta U}{3(1-t\gamma)^2} - \frac{pat^2}{(1+t-t\gamma)^2(1-t\gamma)} - \frac{(a^2 + U)t\theta U}{2(1-t\gamma)^2}.
\]

Since

\[
(p+4)a^4 + pUa^2 = \frac{ap^2(1-t\gamma)}{1+t-t\gamma}
\]

by (6.10), it follows that

\[
\frac{\partial E(w)}{\partial \gamma} > \frac{apt}{4(1-t\gamma)(1+t-t\gamma)} + \frac{tU(a^2 + U)}{4(1-t\gamma)^2} + \frac{\theta atU}{3(1-t\gamma)^2} \\
&\quad - \frac{apt^2}{(1-t\gamma)(1+t-t\gamma)^2} - \frac{tU(a^2 + U)}{2(1-t\gamma)^2}.
\]

Omitting the third term on the right side we conclude that

\[
\frac{\partial E(w)}{\partial \gamma} > \frac{apt}{4(1-t\gamma)(1+t-t\gamma)} \left( 1 - \frac{4t}{1+t-t\gamma} \right) + \frac{(1-2\theta)tU(a^2 + U)}{4(1-t\gamma)^2}.
\]

It remains to observe that all terms on the right side are nonnegative if

\[
1 - 3t - t\gamma \geq 0 \quad \text{and} \quad 0 \leq \theta \leq 0.5.
\]

\[\square\]
Proof of (c): $\partial E(\pi)/\partial \gamma < 0$. Differentiating (6.12), we obtain that

$$\frac{\partial E(\pi)}{\partial \gamma} = p \frac{\partial a}{\partial \gamma} + tE(w) - (1 + t - \gamma) \frac{\partial E(w)}{\partial \gamma}$$

$$= \frac{p^2 t}{3(p + 4)a^2 + pU} \left( \frac{\theta U}{1 - t\gamma} - \frac{pt}{(1 + t - t\gamma)^2} \right)$$

$$+ \frac{t}{4p(1 - t\gamma)} \left[ \left( p + 4 \right) a^4 + 2pUa^2 + p(U)^2 \right]$$

$$- \frac{t(1 + t - t\gamma)}{4p(1 - t\gamma)^2} \left[ \left( p + 4 \right) a^4 + 2pUa^2 + p(U)^2 \right]$$

$$+ \frac{p^2 t}{3(p + 4)a^2 + pU} \left( \frac{pt}{(1 + t - t\gamma)^2} - \frac{\theta U}{1 - t\gamma} \right)$$

$$+ \frac{2t\theta(1 + t - t\gamma)[pUa^2 + p(U)^2]}{4p(1 - t\gamma)^2}$$

$$= \frac{-t}{4p(1 - t\gamma)^2} \left[ t(p + 4)a^4 + 2[t - \theta(1 + t - t\gamma)]pUa^2 \right.$$}

$$+ \left. [t - 2\theta(1 + t - t\gamma)]p(U)^2 \right].$$

Hence,

$$4p(1 - t\gamma)^2 \frac{\partial E(\pi)}{\partial \gamma} = -t^2(p + 4)a^4$$

$$- 2tpa^2U [t - \theta(1 + t - t\gamma)] - tp(U)^2 [t - 2\theta(1 + t - t\gamma)].$$

Our assumption on $\theta$ implies that $t - \theta(1 + t - t\gamma) \geq 0$ and $t - 2\theta(1 + t - t\gamma) \geq 0$, so that $\partial E(\pi)/\partial \gamma < 0$ for all $0 < \theta \leq 1/2$. □

Proof of (d): $\partial E(W)/\partial \gamma < 0$. Differentiating (6.13) with respect to $\gamma$ and using the expression of $\partial E(\pi)/\partial \gamma$ and $\partial E(w)/\partial \gamma$ obtained above, we obtain that

$$\frac{\partial E(W)}{\partial \gamma} = p \frac{\partial a}{\partial \gamma} + \frac{\partial U}{\partial \gamma} + tE(w) - (1 + t - \gamma) \frac{\partial E(w)}{\partial \gamma}$$

$$= \frac{\partial E(\pi)}{\partial \gamma} + t \frac{\partial E(w)}{\partial \gamma} + \frac{\partial U}{\partial \gamma}.$$
It follows that
\[
\frac{\partial E(W)}{\partial \gamma} = \frac{-t}{4p(1-t\gamma)^2} \left[ t(p + 4)a^4 + 2(t - \theta(1 + t - t\gamma))pUa^2 + (t - 2\theta(1 + t - t\gamma))p(\overline{U})^2 \right]
\]
\[
+ \frac{t}{4p(1-t\gamma)^2} \left[ t(p + 4)a^4 + 2tpUa^2 + tp(\overline{U})^2 \right]
\]
\[
+ \frac{p^2 t^2}{(1 + t - t\gamma)[pU + 3(p + 4)a^2]} \left[ \frac{\theta U}{1 - t\gamma} - \frac{pt}{(1 + t - t\gamma)^2} \right]
\]
\[
- \frac{pa^2 + pU t^2 \theta U}{2p(1-t\gamma)^2} \frac{t\gamma U}{(1-t\gamma)}
\]
\[
= \frac{-t}{4p(1-t\gamma)^2} \left[ -2\theta(1 + t - t\gamma)pUa^2 - 2\theta(1 + t - t\gamma)p(\overline{U})^2 \right]
\]
\[
+ \frac{p^2 t^2}{(1 + t - t\gamma)[pU + 3(p + 4)a^2]} \left[ \frac{\theta U}{1 - t\gamma} - \frac{pt}{(1 + t - t\gamma)^2} \right]
\]
\[
- \frac{pa^2 + pU t^2 \theta U}{2p(1-t\gamma)^2} \frac{t\gamma U}{(1-t\gamma)}
\]
\[
= \frac{\theta U(t + t^2 - t^2\gamma)[a^2 + U]}{2(1-t\gamma)^2} - \frac{t^2 \theta U[a^2 + U]}{2(1-t\gamma)^2} - \frac{t\gamma U}{(1-t\gamma)}
\]
\[
+ \frac{p^2 t^2}{(1 + t - t\gamma)[pU + 3(p + 4)a^2]} \left[ \frac{\theta U}{1 - t\gamma} - \frac{pt}{(1 + t - t\gamma)^2} \right]
\]
\[
- \frac{pa^2 + pU t^2 \theta U}{2p(1-t\gamma)^2} \frac{t\gamma U}{(1-t\gamma)}
\]
\[
= \frac{t\theta U[a^2 + U - 2]}{2(1-t\gamma)}
\]
\[
+ \frac{p^2 t^2}{(1 + t - t\gamma)[pU + 3(p + 4)a^2]} \left[ \frac{\theta U}{1 - t\gamma} - \frac{pt}{(1 + t - t\gamma)^2} \right]
\]

Sufficient conditions for this derivative to be negative is that \( \theta < \frac{pt(1-t\gamma)}{U(1+t-t\gamma)^2} \) and that \( a^2 < 2 - \overline{U} \). Using (6.10), the second condition becomes \( \overline{U} < 2 \) and
\[
(p + 4)(2 - \overline{U})^{3/2} + p\overline{U}(2 - \overline{U})^{1/2} > \frac{p^2(1 - t\gamma)}{1 + t - t\gamma}.
\]

It follows that a sufficient condition for the derivative to be negative is that \( 0 < \overline{U} < 2 \) and
\[
\frac{p}{\sqrt{2}(1 + t - t\gamma)} < \overline{U}_0(1 - t\gamma)^{\theta-1} < \frac{pt}{\theta(1 + t - t\gamma)^2}.
\]
## Computation Results for Section 3.1

### Table 1: Simulation of effort, $a$, for $\theta = 0, \bar{U} = 1.5, p = 3$

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.853</td>
<td>0.816</td>
<td>0.782</td>
<td>0.752</td>
<td>0.723</td>
<td>0.696</td>
<td>0.671</td>
<td>0.647</td>
<td>0.625</td>
</tr>
<tr>
<td>0.2</td>
<td>0.852</td>
<td>0.815</td>
<td>0.779</td>
<td>0.747</td>
<td>0.716</td>
<td>0.686</td>
<td>0.658</td>
<td>0.632</td>
<td>0.606</td>
</tr>
<tr>
<td>0.3</td>
<td>0.852</td>
<td>0.813</td>
<td>0.776</td>
<td>0.741</td>
<td>0.708</td>
<td>0.675</td>
<td>0.644</td>
<td>0.614</td>
<td>0.585</td>
</tr>
<tr>
<td>0.4</td>
<td>0.851</td>
<td>0.812</td>
<td>0.773</td>
<td>0.736</td>
<td>0.699</td>
<td>0.663</td>
<td>0.628</td>
<td>0.594</td>
<td>0.559</td>
</tr>
<tr>
<td>0.5</td>
<td>0.851</td>
<td>0.810</td>
<td>0.769</td>
<td>0.729</td>
<td>0.690</td>
<td>0.650</td>
<td>0.610</td>
<td>0.570</td>
<td>0.529</td>
</tr>
<tr>
<td>0.6</td>
<td>0.851</td>
<td>0.808</td>
<td>0.766</td>
<td>0.723</td>
<td>0.679</td>
<td>0.635</td>
<td>0.589</td>
<td>0.541</td>
<td>0.492</td>
</tr>
<tr>
<td>0.7</td>
<td>0.850</td>
<td>0.807</td>
<td>0.762</td>
<td>0.716</td>
<td>0.668</td>
<td>0.617</td>
<td>0.564</td>
<td>0.507</td>
<td>0.445</td>
</tr>
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<td>0.8</td>
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<td>0.805</td>
<td>0.758</td>
<td>0.708</td>
<td>0.655</td>
<td>0.597</td>
<td>0.534</td>
<td>0.465</td>
<td>0.385</td>
</tr>
<tr>
<td>0.9</td>
<td>0.849</td>
<td>0.803</td>
<td>0.753</td>
<td>0.699</td>
<td>0.640</td>
<td>0.574</td>
<td>0.499</td>
<td>0.411</td>
<td>0.305</td>
</tr>
</tbody>
</table>

### Table 2: Simulation of expected wage, $E(w)$, for $\theta = 0, \bar{U} = 1.5, p = 3$

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
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<td>1.279</td>
<td>1.221</td>
<td>1.172</td>
<td>1.131</td>
<td>1.095</td>
<td>1.065</td>
<td>1.038</td>
</tr>
<tr>
<td>0.2</td>
<td>1.444</td>
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<td>1.312</td>
<td>1.263</td>
<td>1.222</td>
<td>1.188</td>
<td>1.160</td>
<td>1.137</td>
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</tr>
<tr>
<td>0.3</td>
<td>1.457</td>
<td>1.397</td>
<td>1.348</td>
<td>1.308</td>
<td>1.276</td>
<td>1.251</td>
<td>1.233</td>
<td>1.221</td>
<td>1.215</td>
</tr>
<tr>
<td>0.4</td>
<td>1.471</td>
<td>1.423</td>
<td>1.385</td>
<td>1.356</td>
<td>1.336</td>
<td>1.323</td>
<td>1.319</td>
<td>1.322</td>
<td>1.334</td>
</tr>
<tr>
<td>0.5</td>
<td>1.486</td>
<td>1.451</td>
<td>1.425</td>
<td>1.408</td>
<td>1.402</td>
<td>1.405</td>
<td>1.419</td>
<td>1.445</td>
<td>1.487</td>
</tr>
<tr>
<td>0.6</td>
<td>1.500</td>
<td>1.479</td>
<td>1.467</td>
<td>1.465</td>
<td>1.475</td>
<td>1.499</td>
<td>1.539</td>
<td>1.601</td>
<td>1.691</td>
</tr>
<tr>
<td>0.7</td>
<td>1.515</td>
<td>1.508</td>
<td>1.512</td>
<td>1.527</td>
<td>1.558</td>
<td>1.608</td>
<td>1.686</td>
<td>1.804</td>
<td>1.984</td>
</tr>
<tr>
<td>0.8</td>
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<td>1.539</td>
<td>1.559</td>
<td>1.595</td>
<td>1.652</td>
<td>1.739</td>
<td>1.873</td>
<td>2.088</td>
<td>2.453</td>
</tr>
<tr>
<td>0.9</td>
<td>1.546</td>
<td>1.571</td>
<td>1.610</td>
<td>1.670</td>
<td>1.759</td>
<td>1.897</td>
<td>2.122</td>
<td>2.520</td>
<td>3.353</td>
</tr>
</tbody>
</table>

### Table 3: Simulation of expected profit, $E(\pi)$, for $\theta = 0, \bar{U} = 1.5, p = 3$

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.999</td>
<td>0.858</td>
<td>0.723</td>
<td>0.594</td>
<td>0.469</td>
<td>0.347</td>
<td>0.228</td>
<td>0.111</td>
<td>-0.004</td>
</tr>
<tr>
<td>0.2</td>
<td>0.997</td>
<td>0.852</td>
<td>0.711</td>
<td>0.573</td>
<td>0.436</td>
<td>0.301</td>
<td>0.166</td>
<td>0.031</td>
<td>-0.105</td>
</tr>
<tr>
<td>0.3</td>
<td>0.996</td>
<td>0.847</td>
<td>0.698</td>
<td>0.550</td>
<td>0.401</td>
<td>0.249</td>
<td>0.095</td>
<td>-0.063</td>
<td>-0.227</td>
</tr>
<tr>
<td>0.4</td>
<td>0.994</td>
<td>0.840</td>
<td>0.685</td>
<td>0.525</td>
<td>0.361</td>
<td>0.191</td>
<td>0.012</td>
<td>-0.176</td>
<td>-0.378</td>
</tr>
<tr>
<td>0.5</td>
<td>0.993</td>
<td>0.834</td>
<td>0.670</td>
<td>0.498</td>
<td>0.317</td>
<td>0.123</td>
<td>-0.086</td>
<td>-0.315</td>
<td>-0.570</td>
</tr>
<tr>
<td>0.6</td>
<td>0.991</td>
<td>0.828</td>
<td>0.654</td>
<td>0.469</td>
<td>0.267</td>
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<td>-0.489</td>
<td>-0.825</td>
</tr>
<tr>
<td>0.7</td>
<td>0.990</td>
<td>0.821</td>
<td>0.638</td>
<td>0.436</td>
<td>0.211</td>
<td>-0.046</td>
<td>-0.349</td>
<td>-0.716</td>
<td>-1.184</td>
</tr>
<tr>
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Table 4: Simulation of expected welfare, $E(W)$, for $\theta = 0, U_0 = 1.5, p = 3$

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Computation Results for Section 4.1

Table 5: Simulation of effort, $a$, for $\theta = 1/4, U_0 = 1.5, p = 3$

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Table 6: Simulation of expected wage, $E(w)$, for $\theta = 1/4, U_0 = 1.5, p = 3$

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Table 7: Simulation of expected profit, $E(\pi)$, for $\theta = 1/4, U_0 = 1.5, p = 3$

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Table 8: Simulation of expected welfare, $E(W)$, for $\theta = 1/4, U_0 = 1.5, p = 3$

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