Rethinking Cointegration and the Expectation Hypothesis of the Term Structure

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Abstract

We show that the expectations hypothesis implies that as the time spread between long and short term yields increases, it becomes increasingly less likely to find that the spread is stationary; even if the two yields are cointegrated. In the data, we find that as the time spread increases, rejection rates for cointegration also increase in line with this theoretical prediction. Our results suggest that cointegration tests may not be appropriate tests of the expectations hypothesis.

Keywords: Cointegration, Term Structure, Expectation Hypothesis

JEL Classification: E32

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Introduction

Expectations of future short term interest rates have long played an important role in explanations of the determination of long term interest rates. In its purest form, the long term interest rate is just the average of current and expected future short term rates. The expectations hypothesis (EH) generalizes this idea to include a risk or liquidity premium in the long term rate. Empirically, both long term and short term rates are not stationary, and are typically found to be integrated of order one. If the risk or liquidity premiums are stationary, then the EH suggests that short term and long term interest rates may be cointegrated. In a seminal paper, Hall et al. (1992) use the EH to motivate the modeling of the term structure of Treasury rates as a cointegrated system. However, evidence on this hypothesis is mixed at best.¹

In this paper, we show that the EH implies that the spread between long and short term rates will converge to a random walk as the term to maturity on the long rate approaches infinity. A somewhat surprising implication of this result is that the EH predicts it will be more difficult to find cointegration as the time spread between rates increases. More specifically, if the spread is used to test for cointegration, then as the time spread between rates increases, it will become increasingly more difficult to reject the null of no cointegration, and so make it more difficult to find the traditional cointegration evidence used to support the EH.

Our analysis begins by formally proving that as the time spread between rates goes to infinity the term spread converges to an infinite order moving average (MA) process identical to the MA representation of a random walk. So, in the limit, the spread will not be stationary, and the null of no cointegration will not be rejected.

The practical importance of this limiting proposition (LP) is an empirical question. To

¹The literature on term structure is vast. The works of Campbell and Shiller (1991), Anderson (1997), Tzavalis and Wickens (1997), and Sarno and Thornton (2003) are closely related to ours.
examine this question, we investigate whether cointegration defined by the spread is more likely to be found in spreads where the time span between the long and the short rates is relatively brief. In particular, we test for bivariate cointegration between combinations of the federal funds rate, the 1-year, 3-year, 5-year, and the 10-year treasury rates. In order to allow for the instability emphasized by Park and Hahn (1999), we carry out a rolling cointegration analysis for each pair of interest rates. We also allow for asymmetric adjustment considered by Enders and Siklos (2001), Clarida et al. (2006) and Sarno et al. (2007). We find that as the time span between rates increase, we are less likely to reject the null of no cointegration.

We next examine an error correction model of interest rates. If the interest rates are cointegrated, then the coefficient on the error correction term should be significantly different than zero. The LP predicts that it will become increasingly less likely to find such a significant coefficient as the time spread increases, and we find support for this implication of the proposition.

The EH predicts that the cointegrating vector will be (1,-1). We utilize the dynamic OLS estimator of Stock and Watson (1993) to estimate the cointegrating vectors, and to test the null that the vector is consistent with the prediction of the EH. In general, it is not. Given this result, we relax the assumption that the cointegrating vector is (1,-1) and reexamine the implications of our theoretical result. In this more general setting, we continue to find support for the the LP, but the support is weaker.
Expectation Hypothesis

Let \( i_{m,t} \) be the yield to maturity of a \( m \)-period pure discount bond that is bought at time \( t \) and matures \( m \) periods ahead. The weakest form of EH can be written as

\[
i_{m,t} - i_{1,t} = \frac{1}{m} \left[ \sum_{i=1}^{m-1} \sum_{j=1}^{i} E_t \Delta i_{1,t+j} \right] + r_m, \tag{1}
\]

where \( E_t \) denotes the expectation conditional on the information at time \( t \), and \( r_m \) represents the (assumed) constant risk premia. The proposition below concerns the limiting behavior of the spread \( i_{m,t} - i_{1,t} \) when the maturity \( m \) rises.

**Limiting Proposition (LP):** Assuming \( \sum_{j=1}^{\infty} E_t \Delta i_{1,t+j} < \infty \). As \( m \to \infty \),

\[
\lim_{m \to \infty} i_{m,t} - i_{1,t} = \sum_{j=1}^{\infty} E_t \Delta i_{1,t+j} + r_m \tag{2}
\]

The assumption \( \sum_{j=1}^{\infty} E_t \Delta i_{1,t+j} < \infty \) is innocuous. It allows the spread \( i_{m,t} - i_{1,t} \) to increase with \( m \), but requires it to remain finite. If even very long term interest rates are finite, this condition will hold. The proof of this proposition is in the appendix.

If we assume, consistent with most of the literature and our empirical results below, that \( E_t \Delta i_{1,t+j} \) are white noise, LP implies that in the limit the spread \( i_{m,t} - i_{1,t} \) has an infinite-order moving average MA(\( \infty \)) representation.\(^3\) Moreover, this moving average has a constant coefficient on each term equal to one. A random walk process has the same MA representation, and therefore in the limit the spread behaves like a random walk process, a nonstationary process.\(^4\)

Intuitively, the nonstationarity is introduced by the double summations in (1). In a single summation of stationary terms, the law of large number implies that multiplying the

\(^2\)See equation (4) of Hall et al., (1992) or equation (3) of Clarida et al. (2006) for a detailed derivation.

\(^3\)In general, \( E_t \Delta i_{1,t+j} \) can possess a high degree of temporal dependence and moderate heteroskedasticity, see Phillips and Durlauf (1986) for instance.

\(^4\)The easiest way to see nonstationary is noticing that the variance of \( \sum_{j=1}^{\infty} E_t \Delta i_{1,t+j} \) in (2) is infinite.
sum by $\frac{1}{m}$ yields stationarity. However, in the double summations $\frac{1}{m}$ does not decay to zero sufficiently, and the necessary scalar for stationarity becomes $\frac{1}{m^{1/2}}$. When $m$ is large, $i_{m,t} - i_{1,t}$ may be represented as the sum of many summed stationary components, and, as a result, the spread behaves like an integrated, that is, a summed process.

LP implies that if we use the spread as the error correction term, then as $m$ rises, the probability of rejecting the null hypothesis of no cointegration will decrease. In the limit, this probability becomes zero. LP may seriously weaken the link between cointegration and EH, and, at least for long time spreads, render cointegration tests of EH inappropriate. Indeed, the failure to find cointegration is predicted by EH in this case.

Data

In this section, we investigate the link between the time spread and the results from cointegration tests. We use monthly observations of the Treasury constant maturity 10-year ($l_{10}$), 5-year ($l_5$), 3-year ($l_3$), and 1-year ($l_1$) yields to maturity, and the effective federal funds rate ($s_f$). The data were downloaded from Federal Reserve Economic Data. The full sample consists of 371 observations from October 1982 to August 2013. Table 1 contains the summary statistics of our data. As we would expect, the yield to maturity increases with the term to maturity. Table 1 also reports results from augmented Dickey-Fuller tests. We find that all series are nonstationary and integrated of order one.

Panel A of Figure 1 plots three of the series: the federal funds rate $s_f$, 3-year rate $l_3$, and the 10-year rate $l_{10}$. Overall, the three series move together. The strength of co-movement of $s_f$ and $l_3$, however, seems stronger than that for $s_f$ and $l_{10}$. In certain periods, such as after year 2009, the link between $s_f$ and $l_{10}$ becomes particularly weak.

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5http://research.stlouisfed.org/fred2/
6Our sample starts in October 1982 since it was then the Federal Reserve ended the policy of targeting monetary aggregates, and resumed targeting interest rates.
Rolling Regression Analysis

Throughout this paper, $y_t$ stands for the long term rate, and $x_t$ the short term rate. Note that the classification of long and short term rates are relative. $l_3$ is the long term rate relative to $s_f$, but the short term rate relative to $l_{10}$. The spread, denoted by $sp$, is given by

$$sp_t = y_t - x_t$$

The spread between the ten year Treasury rate and the federal funds rate is plotted in Panel B of Figure 1.

Tables 2 and 3 present the result of a rolling regression analysis. We adopt the rolling approach for two reasons. First, we allow for instability, perhaps caused by structural changes, in the long-run equilibrium. Second, this method generates enough regression results to obtain meaningful rejection frequency of the cointegration test.

Each window contains 96 consecutive observations, 8 years of data.\footnote{We increase the window size to 120 (10 years of data), and the results are qualitatively unchanged.} This window size is long enough for the cointegration test to have adequate power, but not too long to be contaminated by structural changes. The windows are partially overlapping: the first window uses the first 96 observations; the second window moves one period ahead, and uses the next 96 observations, and so on.

Cointegration Tests

We start with the traditional (linear) cointegration test, which in effect is the unit root test applied to the interest spread $sp_t$. The testing regression is

$$\Delta sp_t = \gamma_0 + \gamma_1 sp_{t-1} + \sum_{i=1}^{p} c_i \Delta sp_{t-i} + v_t.$$  \hspace{1cm} (4)
The null hypothesis is that the bivariate system of \((x_t, y_t)\) is not cointegrated, or equivalently, \(sp_t\) is nonstationary. The null hypothesis is rejected at the 5% level if the t statistic of \(\gamma_1\) is less than -2.86.\(^8\) The number of lags \(p\) is chosen by Ng and Perron (1995) method.\(^9\)

The linear cointegration test imposes the restrictive assumption that the adjustment speed, measured by \(\gamma_1\), is constant. The threshold unit root test of Enders and Granger (1998) and the threshold cointegration test of Enders and Siklos (2001) relax this restriction, and are based on the indicator-augmented regression

\[
\Delta sp_t = \gamma_0 + \gamma_1 sp_{t-1} I(sp_{t-1} < \tau) + \gamma_2 sp_{t-1} I(sp_{t-1} \geq \tau) + \sum_{i=1}^{p} \tilde{c}_i \Delta sp_{t-i} + \tilde{v}_t.
\]

(5)

where the indicator function \(I(\cdot) = 1\) if the event in the parenthesis is true and 0 otherwise. Now the adjustment speeds measured by \(\tilde{\gamma}_1\) and \(\tilde{\gamma}_2\) are possibly asymmetric (i.e., \(\tilde{\gamma}_1 \neq \tilde{\gamma}_2\)). The null hypothesis of no cointegration becomes \(H_0: \tilde{\gamma}_1 = 0; \tilde{\gamma}_2 = 0\). We try two threshold values: \(\tau = 0\), and \(\tau = \text{sample mean of } sp_{t-1}\) (unreported). The results are largely the same.

The percentages of times that the hypothesis of no cointegration cannot be rejected are reported in Table 2. For the spread between the one year Treasury rate and the federal funds rate, the linear cointegration test fails to reject the null of no cointegration 80% of the time. For the spread between the three year Treasury and the federal funds rate, the rejection rate increases to 90%. LP predicts that this rejection rate increases as the time spread increases and Table 2 reveals this pattern. The threshold test also produces this pattern.

\(^8\)Because the cointegrating vector (1,-1) is predetermined, the cointegration test follows the Dickey-Fuller distribution asymptotically.

\(^9\)A well known fact is that the outcome of the unit root test and cointegration test can be sensitive to the lag number \(p\). Accordingly, we redo the tests use the popular method of minimizing the Akaike information criterion (AIC) to select \(p\). We find the same pattern.
Error Correction Models

An alternative method to investigate the cointegration of short term and long term rates is to estimate the error correction model. We estimate

$$\Delta y_t = \alpha_{01} + \alpha_1^* s_{p_t-1} + \sum_{i=1}^P c_i \Delta y_{t-i} + \sum_{i=1}^P d_i \Delta x_{t-i} + v_{1t},$$  \hspace{1cm} (6)

According to LP the spread will behave increasingly like a nonstationary process as the time spread rises. This property, in turn, implies that it will become increasingly less likely to find a significant coefficient on $\alpha_1^*$ in (6) as the time spread increases. Table 3 reports the rejection rates for the significance of the error correction coefficient.$^{10}$ For spreads with the federal funds rate, the rejection rate increases from 56% for the one year spread, to 85% for the three year spread, and then settles in the low to mid 90% range for the longer time spreads. For the Treasury spreads, with one exception, the rejection rates are all in the low to mid 90% range.

Cointegrating Vectors

The cointegrating vector specified by the spread $s_{p_t}$ is restricted to be $(1, -1)$. We can test this restriction by considering the cointegrating regression

$$y_t = \beta_0 + \beta_1 x_t + e_t.$$ \hspace{1cm} (7)

We estimate the cointegrating regression (7) using rolling windows. Panel A of Figure 2 plots the series of $\hat{\beta}_1$ for differing long term rates when the short term rate is the federal funds rate. There are two main findings from Panel A. First, as the time spread increases, $\hat{\beta}_1$ gets smaller, and farther away from one. This increasing deviation from one is consistent

$^{10}$The t statistic of $\alpha_1^*$ follows a nonstandard distribution under the null hypothesis, so we ran simulations to obtain the critical values.
with LP. When $\hat{\beta}_1$ is unity, $\hat{e}_t$, which is the OLS residual, reflects fluctuations in the term spread. As the spread behaves more and more like nonstationary series, then $\hat{\beta}_1$ will move away from unity since OLS estimation will, by its nature, attempt to maximize the chance of stationarity in the residual.

Second, there are clearly two periods that show evidence of instability, 1982-1984 and 1990-1994. It is beyond the scope of this paper to investigate the source of this instability, but we do note that both periods occur near the trough of a business cycle. Also, the earlier period follows shortly after the Volcker disinflation, while the later period follows the creation of the Resolution Trust Corporation. These conclusions carry over to Panel B where term spreads between Treasury securities are plotted.

Panels C and D of Figure 2 plot the serial-correlation adjusted t statistics for the null hypothesis

$$H_0 : \beta_1 = 1$$

using the dynamic OLS estimator of Stock and Watson (1993). Stock and Watson propose the following regression

$$y_t = \tilde{\beta}_0 + \tilde{\beta}_1 x_t + \sum_{i=-p}^{p} k_i \Delta x_{t-i} + u_t. \tag{8}$$

This regression includes the leads and lags of $\Delta x_t$ as additional regressors so that a modified t statistic asymptotically follows the normal distribution. Here, we let $p = 4$, and estimate the long run variance (spectral density at frequency zero) of $u_t$ by running an auxiliary autoregression for the residual $\hat{u}_t$.\footnote{See equation (19.3.27) of Hamilton (1994) for details. We also try various lag $p$ for the dynamic OLS regression (8), and we re-estimate the long run variance of $u_t$ using the method of Newey and West (1987) with truncation at 4 or 8. There is no qualitative change.}

For most spreads and for most periods, the t statistics are outside the (-1.96, 1.96) bound. Nearly all t statistics are less than -1.96 after 1994, while between 1982 and 1985, $\hat{\beta}_1$ is insignificantly different from unity.
More General Cointegration

Given the results of the previous section, we relax the restriction that the cointegrating vector is (1,-1). We can rationalize this generalization by dropping the assumption that the risk premium is constant. Instead, we assume that this premium may be non-stationary and correlated with the interest rate. If this is the case, then the spread is no longer the correct variable of interest and we must allow for a more general form of cointegration. From (7) we now estimate the error correction term as

$$\hat{e}_t = y_t - \hat{\beta}_0 - \hat{\beta}_1 x_t$$

We plot the estimated error correction term $\hat{e}_t$ and the spread between the ten year Treasury and the federal funds rate in Panel B of Figure 1. There are a number of periods, for example after 2009 and in the early part of the sample, where the spread and $\hat{e}_t$ clearly differ.

Table 4 reports the percentage of times that we fail to reject the null of no cointegration in the more general setting applying the Engle-Granger and Enders-Siklos tests to $\hat{e}_t$. Comparing Table 4 to Table 2, it is interesting to note that the rejection rates are smaller for nearly every spread, and more importantly, the pattern of increasing percentages as the time spread increases becomes less clear. Table 5 reports the results for estimating the more general error correction model

$$\Delta y_t = \alpha_{02} + \alpha_{2} \hat{e}_{t-1} + \sum_{i=1}^{p} c_{2i} \Delta y_{t-i} + \sum_{i=1}^{p} d_{2i} \Delta x_{t-i} + v_{2t}.$$  \hspace{1cm} (10)

Again, the story is much the same for $\hat{e}_t$ as it is for the spread. Once we allow for a potentially nonstationary risk premium, we find less evidence against EH.

Both (6) and (10) assume that the error correction term is stable. More specifically, that

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\footnote{The critical values are simulated under the null of no cointegration.}
the error correction term is always the spread or always \( \hat{e}_t \). The varying values of \( \hat{\beta}_1 \) shown in Figure 2 suggest that sometimes the true error correction term is the spread and sometimes it is \( \hat{e}_t \). This is possible if the degree to which the markets of federal funds and Treasury bonds are segmented varies over time, or, if the risk premium is sometimes stationary and sometimes nonstationary.\(^{13}\) We propose an augmented error correction model (AECM) that allows this possibility. It is given by

\[
\Delta y_t = \alpha_{03} + \alpha_1 sp_{t-1} + \alpha_2 \hat{e}_{t-1} + \sum_{i=1}^p c3_t \Delta y_{t-i} + \sum_{i=1}^p d3_t \Delta x_{t-i} + v3_t
\]  

(11)

This new model has two merits. First, it compares the significance of \( sp \) and \( \hat{e} \) directly in a single model. Second, it allows for a time-varying error correction term. That is, it can accommodate the possibility that in one period \( sp \) is significant, and so is the appropriate error correction term, and in another period \( \hat{e} \) is significant, and so it is the appropriate correction term.\(^{14}\) Table 6 reports the results for the augmented model. When the federal funds rate is the short rate, 66% of the time the augmented model failed to reject the null that the coefficient on the error correction term equaled zero. For a three year time spread and above, the model failed to reject the same null more than 80% of the time. For Treasury spreads, the rejection rates are high for all time spreads and relatively stable.

Notice that all the percentages in Table 6 are less than those in Table 5, and with only two exceptions, are less than those in Table 3. The augmented model provides the strongest evidence of cointegration. Put differently, the traditional error correction model may fail to find cointegration due to the restriction that the error correction term is time-invariant.

It is interesting to compare \( \alpha_1 \) to \( \alpha_2 \) as it tells us which error correction term is driving the adjustment. Figure 3 plots the series of \( t \) values of \( \alpha_1 \) (Panel A) and \( \alpha_2 \) (Panel B) in the rolling augmented error correction model (11), where \( x_t = s_f \). It is clear that \( \alpha_1 \) tends to be

\(^{13}\)For instance, Panel A of Figure 2 suggests that the risk premium may be nonstationary in early 1990s.

\(^{14}\)If \( \hat{e} \) and \( sp \) are similar, then both can be significant.
significant when $y_t = l_1$; whereas $\alpha_2$ be significant when $y_t = l_{10}$. This finding is consistent with Figure 2. Overall, Figures 2 and 3 indicate that, when the difference between maturity is large, the adjustment can be driven mostly by the estimated error correction, which differs significantly from the spread.

**Conclusion**

Cointegration tests have often been used as tests of the EH. We show that as the time spread between long term and short term rates increases, the yield spread behaves more and more like a random walk. As a result, it becomes less likely to reject the null hypothesis of no cointegration as the time spread increases. This implies that even if the EH holds, we may still fail to find cointegration, especially as the time spread between rates increases. In short, declining rejection rates of cointegration tests is consistent with the expectation hypothesis of the term structure of interest rates.

We show, using a variety of approaches, that it is more likely to fail to reject the null of no cointegration as the time spread increases when the error correction term is restricted to equal the spread. If we relax the assumption that the risk premium is always stationary, the EH allows for a more general error correction term. We propose a new augmented error correction model, in which the spread and estimated error correction term are both included. We can interpret this model as allowing for the stationarity of the risk premium to vary over time. We find some support for this model in the data, and it suggests the implications of relaxing the assumption that the stationarity of the risk premium is time-invariant be investigated in the future.
Appendix: Proof of Limiting Proposition

Rearranging (1) leads to

\[ i_{m,t} - i_{1,t} = \frac{1}{m} \left[ \sum_{i=1}^{m-1} \sum_{j=1}^{i} E_t \Delta i_{1,t+j} \right] + r_m \] (12)

\[ = \frac{1}{m} \left[ E_t \Delta i_{1,t+1} + (E_t \Delta i_{1,t+1} + E_t \Delta i_{1,t+2}) + \ldots \right] + r_m \] (13)

\[ = \frac{1}{m} \left[ (m-1)E_t \Delta i_{1,t+1} + (m-2)E_t \Delta i_{1,t+2} + \ldots + E_t \Delta i_{1,t+m-1} \right] + r_m \] (14)

\[ = \sum_{j=1}^{m-1} \left( 1 - \frac{j}{m} \right) E_t \Delta i_{1,t+j} + r_m \] (15)

\[ = \sum_{j=1}^{m-1} E_t \Delta i_{1,t+j} - \frac{1}{m} \sum_{j=1}^{m-1} j E_t \Delta i_{1,t+j} + r_m \] (16)

By assumption \( \sum_{j=1}^{m-1} E_t \Delta i_{1,t+j} \) is finite, and so is a Cauchy sequence. It follows that (e.g., page 438 of Hayashi (2000))

\[ \lim_{m \to \infty} \frac{1}{m} \sum_{j=1}^{m-1} j E_t \Delta i_{1,t+j} = 0. \] (17)

The proof is done after taking limits on both sides of (16).
References


Table 1: Summary of Full Sample (1982:10-2013:8)

<table>
<thead>
<tr>
<th>Series</th>
<th>n</th>
<th>Mean</th>
<th>SE</th>
<th>Min</th>
<th>Max</th>
<th>ADF</th>
<th>ADF(\Delta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_f)</td>
<td>371</td>
<td>4.58</td>
<td>2.98</td>
<td>0.07</td>
<td>11.64</td>
<td>-1.69</td>
<td>-5.12**</td>
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<tr>
<td>(l_1)</td>
<td>371</td>
<td>4.69</td>
<td>2.95</td>
<td>0.10</td>
<td>12.08</td>
<td>-1.84</td>
<td>-4.73**</td>
</tr>
<tr>
<td>(l_3)</td>
<td>371</td>
<td>5.29</td>
<td>2.99</td>
<td>0.33</td>
<td>13.18</td>
<td>-1.25</td>
<td>-6.32**</td>
</tr>
<tr>
<td>(l_5)</td>
<td>371</td>
<td>5.67</td>
<td>2.86</td>
<td>0.62</td>
<td>13.48</td>
<td>-1.31</td>
<td>-6.69**</td>
</tr>
<tr>
<td>(l_{10})</td>
<td>371</td>
<td>6.17</td>
<td>2.62</td>
<td>1.53</td>
<td>13.56</td>
<td>-1.87</td>
<td>-5.79**</td>
</tr>
</tbody>
</table>

Note:
a. \(n\) is the sample size; \(\text{Mean}\) is the sample mean; \(\text{SE}\) is the standard error; \(\text{Min}\) is the minimum; \(\text{Max}\) is the maximum.
b. \(\text{ADF}\) is the augmented Dickey-Fuller t test applied to the level.
c. \(\text{ADF}\_\Delta\) is the augmented Dickey-Fuller t test applied to the first difference.
d. For the \(\text{ADF}\) test, the number of lag \(p\) is chosen by the method of Ng and Perron (1995). All testing regressions includes a constant, and no trend.
e. ** denotes rejecting the null hypothesis of nonstationarity at the 5% level.

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Table 2: % Fail to Reject No Cointegration Hypothesis Using \(sp_t\)

<table>
<thead>
<tr>
<th>FF rate is short rate(^a)</th>
<th>Treasury spreads(^b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spread</td>
<td>T1-FF</td>
</tr>
<tr>
<td>Linear</td>
<td>80</td>
</tr>
<tr>
<td>Threshold</td>
<td>68</td>
</tr>
</tbody>
</table>

Note:
a. Each cell below the label reports the test outcome when the spread is between a Treasury yield and the federal funds rate. T1 is the one year, T3 is the three year rate, and so on.
b. Each cell below the label reports the test outcome for differing Treasury spreads. The spread T3-T1 is the spread between the three year and one year Treasury rates, T5-T3 is the spread between the five year and three year rate, and so on.
Table 3: % Fail to Reject Zero Error Correction Term Hypothesis Using $sp_t$

<table>
<thead>
<tr>
<th>Spread</th>
<th>T1-FF</th>
<th>T3-FF</th>
<th>T5-FF</th>
<th>T10-FF</th>
<th>T3-T1</th>
<th>T5-T3</th>
<th>T5-T1</th>
<th>T10-T5</th>
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<td>94</td>
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<td></td>
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<tr>
<td>T3-FF</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>92</td>
<td>94</td>
<td>81</td>
<td>90</td>
<td>93</td>
</tr>
</tbody>
</table>

Note:
a. The number in each cell is the rate that the hypothesis $H_0: \alpha_1^* = 0$ fails to be rejected at 5% level in regression (6).

Table 4: % Fail to Reject No Cointegration Hypothesis Using $\hat{\epsilon}_t$

<table>
<thead>
<tr>
<th>Spread</th>
<th>T1-FF</th>
<th>T3-FF</th>
<th>T5-FF</th>
<th>T10-FF</th>
<th>T3-T1</th>
<th>T5-T3</th>
<th>T5-T1</th>
<th>T10-T5</th>
<th>T10-T3</th>
<th>T10-T1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear$^a$</td>
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<td>53</td>
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<td>76</td>
<td>73</td>
<td>87</td>
<td>75</td>
<td>98</td>
<td>91</td>
<td>89</td>
</tr>
<tr>
<td>Threshold$^b$</td>
<td>74</td>
<td>62</td>
<td>71</td>
<td>80</td>
<td>82</td>
<td>88</td>
<td>83</td>
<td>99</td>
<td>92</td>
<td>85</td>
</tr>
</tbody>
</table>

Note:
a. Each cell is the percent that the null hypothesis of no cointegration cannot be rejected after applying the Engle-Granger Test to the estimated error correction term $\hat{\epsilon}_t$.
b. Each cell is the percent that the null hypothesis of no cointegration cannot be rejected after applying the Enders-Siklos Test to the estimated error correction term $\hat{\epsilon}_t$.

Table 5: % Fail to Reject Zero Error Correction Term Hypothesis Using $\hat{\epsilon}_t$

<table>
<thead>
<tr>
<th>Spread</th>
<th>T1-FF</th>
<th>T3-FF</th>
<th>T5-FF</th>
<th>T10-FF</th>
<th>T3-T1</th>
<th>T5-T3</th>
<th>T5-T1</th>
<th>T10-T5</th>
<th>T10-T3</th>
<th>T10-T1</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1-FF</td>
<td>70</td>
<td>93</td>
<td>94</td>
<td>93</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T3-FF</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>93</td>
<td>79</td>
<td>92</td>
<td>86</td>
<td>90</td>
<td>95</td>
</tr>
</tbody>
</table>

Note:
a. The number in each cell is the rate that the hypothesis $H_0: \alpha_2^* = 0$ fails to be rejected at 5% level in regression (10).

Table 6: % Fail to Reject Zero Error Correction Term Hypothesis Augmented Error Correction Model

<table>
<thead>
<tr>
<th>Spread</th>
<th>T1-FF</th>
<th>T3-FF</th>
<th>T5-FF</th>
<th>T10-FF</th>
<th>T3-T1</th>
<th>T5-T3</th>
<th>T5-T1</th>
<th>T10-T5</th>
<th>T10-T3</th>
<th>T10-T1</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1-FF</td>
<td>66</td>
<td>85</td>
<td>86</td>
<td>84</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T3-FF</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>84</td>
<td>55</td>
<td>81</td>
<td>71</td>
<td>76</td>
<td>86</td>
</tr>
</tbody>
</table>

Note:
a. Each cell is the rate that neither the hypothesis $H_0: \alpha_1 = 0$ nor the hypothesis $H_0: \alpha_2 = 0$ is rejected at 5% level in regression (11).
Figure 1: Time Series Plot

Panel A: SF, L3, and L10

Panel B: Two Error Correction Terms

Note:

a. Panel A plots the federal funds rate $s_f$ (solid line), the 3-year treasury bond rate $l_{10}$ (dotted line) and the 10-year treasury bond rate $l_{10}$ (dash line).

b. Panel B plots the centered interest rate spread $sp = l_{10} - s_f$ (solid line), and the OLS residual of regressing $l_{10}$ on a constant and $s_f$ (dotted line).
Figure 2: Estimated Beta1

Panel A: X=SF
Y=L1 Y=L3 Y=L5 Y=L10

Panel B: X=L1
Y=L3 Y=L5 Y=L10

Panel C: X=SF
T test Beta1=1

Panel D: X=L1
T test Beta1=1

Note:

a. Panel A plots the series of estimated $\beta_1$ in (7) using $x_t = s_f$, $y_t = l_1, l_3, l_5, l_{10}$. The regression is fitted for the rolling window that starts at $t_0$ and ends at $t_0 + w - 1$, where the window size is $w = 96$, and $t_0 = (1, \ldots, 371 + 1 - w)$.

b. Panel B plots the series of estimated $\beta_1$ in (7) using $x_t = l_1$, $y_t = l_3, l_5, l_{10}$.

c. Panel C plots the serial-correlation adjusted $t$ statistic for $H_0: \beta_1 = 1$ in (8) using $x_t = s_f$.

d. Panel D plots the serial-correlation adjusted $t$ statistic for $H_0: \beta_1 = 1$ in (8) using $x_t = l_1$. 
Figure 3: T Values of Alphas in Rolling AECMs

Panel A: Alpha1

Panel B: Alpha2

Note:

a. Panel A plots the series of t value of $\alpha_1$ in (11) using $x_t = s_f$, $y_t = l_1, l_3, l_{10}$. The regression is fitted for the rolling window that starts at $t_0$ and ends at $t_0 + w - 1$, where the window size is $w = 96$, and $t_0 = (1, \ldots, 371 + 1 - w)$.

b. Panel B plots the series of t value of $\alpha_2$ in (11).