APPENDIX A
Basic Mathematical Tools

SOLUTIONS TO PROBLEMS

A.1 (i) The average monthly housing expenditure is $566.

(ii) The two middle numbers are 480 and 530; when these are averaged, we obtain the median monthly housing expenditure as 505, or $505.

(iii) If monthly housing expenditures are measured in hundreds of dollars, the average and median monthly housing expenditures are 5.66 and 5.05, respectively.

(iv) The average increases to $586 while the median is unchanged ($505).

A.2 (i)

This is just a standard linear equation with intercept equal to 3 and slope equal to .2. The intercept is the number of classes missed by a student who lives on campus.

(ii) The average number of classes missed for someone who lives five miles away is $3 + .2(5) = 4$ classes.

(iii) The difference in the average number of classes missed for someone who lives 10 miles away and someone who lives 20 miles away is $10(.2) = 2$ classes.

A.3 If $price = 15$ and $income = 200$, $quantity = 120 - 9.8(15) + .03(200) = -21$, which is nonsense. This shows that linear demand functions generally cannot describe demand over a wide range of prices and income.
A.4 (i) The percentage point change is $5.6 - 6.4 = -.8$, or an eight-tenths of a percentage point decrease in the unemployment rate.

(ii) The percentage change in the unemployment rate is $100[(5.6 - 6.4)/6.4] = -12.5\%$.

A.5 The majority shareholder is referring to the percentage point increase in the stock return, while the CEO is referring to the change relative to the initial return of 15%. To be precise, the shareholder should specifically refer to a three percentage point increase.

A.6 (i) The exact percentage by which Person B’s salary exceeds Person A’s is $100[42,000 - 35,000]/35,000 = 20\%$.

(ii) The approximate proportionate change is $\log(42,000) - \log(35,000) \approx .182$, so the approximate percentage change is $18.2\%$. [Note: $\log(\cdot)$ denotes the natural log.]

A.7 (i) When $\text{exper} = 0$, $\log(\text{salary}) = 10.6$; therefore, $\text{salary} = \exp(10.6) \approx 40,134.84$. When $\text{exper} = 5$, $\text{salary} = \exp[10.6 + .027(5)] \approx 45,935.80$.

(ii) The approximate proportionate increase in $\text{salary}$ when $\text{exper}$ increases by five years is $.027(5) = .135$, so the approximate percentage change is $13.5\%$.

(iii) $100[(45,935.80 - 40,134.84)/40,134.84] \approx 14.5\%$, so the exact percentage increase is about one percentage point higher.
(ii) Compared with a linear function, the function

\[
yield = 120 + .19 \sqrt{\text{fertilizer}}
\]

has a diminishing effect, and the slope approaches zero as \textit{fertilizer} gets large. The initial pound of fertilizer has the largest effect, and each additional pound has an effect smaller than the previous pound.

**A.10** (i) The value 45.6 is the intercept in the equation, so it literally means that if \textit{class} = 0, then the \textit{score} is 45.6. Of course, \textit{class} = 0 can never happen. And values close to zero would rarely if ever occur, except in very small schools. So, by itself, 45.6 is not of much interest. But it must be accounted for to use the equation to obtain \textit{score} for sensible values of \textit{class}.

(ii) We use calculus to obtain the optimal class size:

\[
class^* = 0.082 / [2(0.000147)] \approx 278.91.
\]

Rounded to the nearest integer, the optimal class size is 279 students. When we plug this into the equation for \textit{score}, we get the largest achievable score:

\[
\textit{score}^* = 45.6 + 0.082(279) - 0.000147(279^3) \approx 57.04.
\]

(iii) The following graph shows the solution rounded to the nearest integer: